所別：大氣物理研究所碩士班 一般生 科目：應用數學

1. Diagonalization of the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$.

(10%)

2. Transform $Q = x_1^2 + 6x_1x_2 + 9x_2^2 = 10$ to the principal (new) axes and determine the new axes using the old axes (x_1, x_2).

(10%)

3. (a) Find the Fourier integral representation of the function
$$f(x) = \begin{cases} k & if \ |x| < 1 \\ 0 & if \ |x| > 1 \end{cases}.$$

(10%)

(b) From (a), show that
$$\int_0^\infty \frac{\cos x \sin x}{x} \, dx = \frac{\pi}{4}.$$

(5%)

4. Solve the heat equation in a finite bar of length L
$$u_t = c^2 u_{xx} \quad (0 \le x \le L, t > 0)$$
with the following boundary conditions
$$u_x(0,t) = 0 \quad and \quad u_x(L,t) = 0,$$
and the initial condition
$$u(x, 0) = f(x),$$
where c and k are real constants.

(15%)

5. Solve the following Bernoulli differential equation.
$$y' + 2y = y^2$$

(10%)

6. Solve the following initial value problem,
$$y_1'' + \omega_0^2 y_1 = -\alpha(y_1 - y_2)$$

$$y_2'' + \omega_0^2 y_2 = \alpha(y_1 - y_2)$$

And $y_1(0) = a_1$, $y_1'(0) = a_2$, $y_2(0) = b_1$, and $y_2'(0) = b_2$.

(15%)

7. Find the eigenvalues and eigenfunctions of the following problem
$$(xy')' + \lambda x^{-1} y = 0, \quad y(1) = 0, \ y'(e) = 0$$

(15%)

8. Find the inverse Laplace transform of the follwing function
$$\frac{s^4 + 3(s+1)^3}{s^4(s+1)^3}$$

(10%)