

1. 解釋下列名詞並舉例說明之。

- a. 史登-路易維爾問題(Sturm-Liouville problem)
- b. 單位階梯函數(unit step function)
- c. 泰勒級數(Taylor series)
- d. 位理論(potential theory)
- e. 強迫振盪(forced oscillation)

(15%)

2. 請解下列之初始值問題(initial value problem)。

$$x^2 y'' + xy' - y = 16x^3 \quad y(1) = -1 \quad y'(1) = 11$$

(15%)

3. 請利用拉普拉斯轉換解下列之積分方程式(integral equation)。

$$y = 2t - 4 \int_0^t y(\tau)(t - \tau) d\tau$$

(15%)

4. 請解出下列行列式(determinant)的值。

$$\begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{vmatrix}$$

(10%)

5. 何謂傅利葉級數(Fourier series) 和傅利葉積分(Fourier integral)? 請說明它們之間的差異與應用範圍。何謂傅利葉轉換(Fourier transform)? 請說明與比較傅利葉轉換與拉普拉斯轉換之間的差異與應用範圍。請解出下列週期函數(periodic function)的傅利葉級數。

$$f(x) = \pi - x, \quad 0 < x < 2\pi$$

(20%)

6.

- (a) Write down the definitions of gradient, divergence, and curl.
- (b) Use a physical phenomenon or daily experiences to explain the meanings of these quantities.

(10%)

7. If $\vec{F} = \nabla \times \vec{A}$, compute the integration of \vec{F} over any closed surface.

(5%)

8. Let $\vec{F} = 2xy^4 \vec{i} + 4x^2y^3 \vec{j}$. If \vec{F} is integrated along curve $y = 3x^2 + 2$ from