國立中央大學106學年度碩士班考試入學試題

所別： 大氣科學學系大氣物理碩士班 不分組(一般生)
大氣科學學系大氣物理碩士班 不分組(在職生)

科目： 應用數學

本科考試禁用計算器

＊請在答案卷內作答

參考用

1. Please explain and compare the following terms.

(5%)a. Power series method and extended power series method (Frobenius method);

(5%)b. Orthogonality of vectors and orthogonality of functions;

(5%)c. Laplace transform and Fourier transform;

(5%)d. Unit step function and Dirac delta function;

(20%)

2. Find orthogonal trajectories of the following function

$$x^2+(y-c)^2=c^2$$

(10%)

3. Solve the following initial value problem.

$$(D^2+4D+5I)y=e^{-t}\cos t \qquad y(0)=0,\ y'(0)=1$$

(15%)

4. Is the given matrix Hermitian? Skew-Hermitian? Unitary? Find tits eigenvalues and eigenvectors.

$$\begin{bmatrix} 0 & 2+2i & 0 \\ 2-2i & 0 & 2+2i \\ 0 & 2-2i & 0 \end{bmatrix}$$

(15%)

5. Evaluate Let $\int_C \mathbf{F}\cdot\mathbf{r}'ds$, where C is the circle $x^2+y^2=4$, $z=-3$, oriented counterclockwise as seen by a person standing at the origin, and, with respect to right-handed Cartesian coordinates, $\mathbf{F}=y\mathbf{i}+xz^3\mathbf{j}-zy^3\mathbf{k}$.

(15%)

6. Please find the corresponding Fourier series of the following function.

$$f(x)=x+\pi, \quad \text{if } -\pi<x<\pi \quad \text{and} \quad f(x+2\pi)=f(x)$$

(15%)

7. Let $\bar{u}$ and β be constant, find a plane wave solution of the following equation,

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+\beta\frac{\partial\psi}{\partial x}=0$$

(10%)