國立中央大學104學年度碩士班考試入學試題

所別：大氣科學學系大氣物理碩士班 不分組(一般生)　科目：應用數學　共 1 頁 第 1 頁
大氣科學學系大氣物理碩士班 不分組(在職生)

本科考試禁用計算器　　＊請在答案卷（卡）內作答

1. Please explain and compare the following terms.
   a. Divergence and Curl;
   b. Stokes theorem and divergent theorem;
   c. Skew-symmetric and Orthogonal matrices;
   d. Linear and nonlinear differential equations;
   e. Fourier series and Fourier integral.

   (20%)

2. Solve the following initial value problem.

   $(x^2D^2+xD-I)y=16x^3 \qquad y(1)=-1, \qquad y'(1)=1$

   (15%)

3. Find the eigenvalues and eigenfunctions of the following Sturm-Liouville problem. Verify orthogonality.

   $y''-2y'+(\lambda+1)y=0, \qquad y(0)=0, \qquad y(1)=0$

   (15%)

4. Find the inverse by Gauss-Jordan method.

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix}.$$

   (10%)

5. Solve the following equation by Laplace transform.

$$x\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}=xt, \qquad w(x,0)=0 \text{ if } x\geq 0 \text{ and } w(0,t)=0 \text{ if } t\geq 0$$

   (15%)

6. Please find the corresponding Fourier series of the following function.

$$f(x)=\begin{cases} -k & \text{if } -2<x<0 \\ k & \text{if } 0<x<2 \end{cases} \qquad period=4$$

   (10%)

7. Find a basis of eigenvectors and diagonalize the following matrix.

$$\begin{bmatrix} -12 & 22 & 6 \\ 8 & 2 & 6 \\ -8 & 20 & 16 \end{bmatrix}.$$

   (15%)