所別：機械工程學系碩士班 丙組(熱流) 科目：流體力學

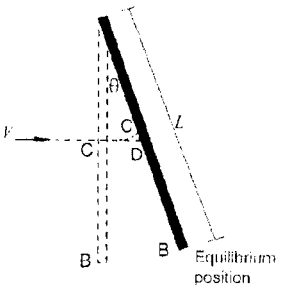
1. 非洲土人常用吹箭來打獵，今有一吹箭筒長爲 2m，內直徑爲 3cm，吹箭重 0.5 N, 其側邊(和箭筒的接觸)面積爲 1500 mm^2。假設吹箭和箭筒中間有 0.01mm 的間隙，間隙中有空氣和獵人吹氣的混合氣体，可當作減少摩擦力之潤滑劑，其黏滯係數爲 $3x10^{-5}$ $N·s/m^2$, 而土人的吹氣壓力一直維持 5 kPa gauge，求吹箭離開箭筒時可達的速度爲何? (15 %)

2. 因爲表面張力的作用，鋼針可浮在水面上。假設鋼針浮在水面上時，恰有一半体積在水面下，而表面張力的方向爲向上，其大小爲 1000N/m，試問鋼針最大容許的直徑爲何? 鋼針的比重爲 7.8，水的密度爲 1000kg/m^3，浮力的作用可忽略不計。 (15%)

3. (a) Show that the volumetric dilatation rate (the rate of volume change per unit volume) of a fluid is equal to $\nabla \cdot \mathbf{V}$ (divergence of the fluid velocity vector). (7%)
   (b) Consider an incompressible, plane potential flow, where a source is located at the origin, $(x, y) = (0,0)$. Do you agree that one can apply $\nabla \cdot \mathbf{V} = 0$ in this flow field with the exception at the origin? Prove and explain your answer. (10%)

4. The volume V of a drop of liquid that forms at the end of a tube before it falls under the action of gravity, depends on the tube diameter d, coefficient of surface tension σ, and the specific weight γ. Perform dimensional analysis for V. (8%)

5. An air jet of density ρ strikes a hinged vertical plate at its center. The jet velocity is V, jet cross-sectional area A, and the plate length L.
   (a) Use control volume analysis to determine the angle of deflection θ as shown in the figure. (5%)
   (b) Determine the magnitude of force F that is needed to apply at the lower edge of the plate to keep the plate vertical. (5%)



6. Please define and answer briefly the following questions. (total 8 %)
   (1) What is the ratio of inertia force to viscous force? (a) Froude number; (b) Reynolds number; (c) Mach number; (d) Strouhal number. (one answer) (1 %)
   (2) What are the conditions for the fully developed laminar flow of a circular pipe with the radius r in the cylindrical coordinate? (a) $v_r = v_r(r)$; (b) $v_\theta = v_\theta(r)$; (c) $v_z = v_z(r)$; (d) $v_z = 0$; (e) $v_\theta = 0$. (more than one answer) (2 %)
   (3) How to determine the Darcy friction factor from the moody chart when the flow is laminar? (a) $32/Re$; (b) $64/Re$; (c) $32/\sqrt{Re}$; (d) $64/\sqrt{Re}$. (one answer) (1 %)
   (4) What are the speed of sound, a, and the Mach number, Ma? (2 %)
   (5) The energy equation for isentropic flow is $h + u^2/2 = h_0$. Please show that: (2 %)

$$a^2 + \frac{\gamma - 1}{2} u^2 = a_0^2 .$$

注意：背面有試題

7. Consider an incompressible, Newtonian fluid in a two-dimensional flow field. (total 14 %)

(a) Use the similitude argument to obtain the non-dimensional continuity and momentum equations. (8 %)

The dimensional continuity and momentum equations are given as follows.

continuity: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

x-Mom: $\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$

y-Mom: $\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\rho g+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$

Hint: Use the following reference variables to non-dimensionalize u, v, p, x, y and t, for examples, the characteristic velocity (V) for u and v, the characteristic pressure (P_o) for p, the characteristic length (L) for x and y, and the characteristic time (τ) for t.

(b) Identify each term in the above non-dimensional momentum equations with the appropriate forces using the following corresponding numerals: (1) for inertia (local) force; (2) for inertia (convective) force; (3) for pressure force; (4) for gravitational force; (5) for viscous force. (3 %)

(c) Write down non-dimensional parameters including the Strouhal, Euler, Reynolds, and Froude numbers in the above non-dimensional momentum equations. (3 %)

8. A viscous liquid flows steadily down an inclined plane surface with an angle θ, having a fully developed laminar film of thickness h, as shown below. (total 13 %)

(a) Please write down the governing equations for this flow field by simplifying the Navier-Stokes equations given in Problem 7. (5%)

(b) Write down the appropriate boundary conditions and the liquid velocity profile $u(y)$? (5 %)

(c) Write down the shear stress distribution, τ_{yx}. (3 %)