

所別：機械工程學系碩士班丁、戊組

科目：工程數學(含程式設計)

Ordinary Differential Equation (25%)

1. (a) Please use Laplace Transform to solve the ordinary differential equation of $y'' + 25y = 5\delta(t - \pi)$, with initial conditions of $y(0) = 3$ and $y'(0) = 0$. Note that δ is the Dirac delta function. (12%)

(b) Calculate the values of $y(\pi/2)$ and $y(2\pi)$. (3%)

2. Solve $y' + y = -2x/y$ with initial condition of $y(0) = 2$. (10%)

Linear Algebra & Vector Calculus (25%)

3. For the linear system $Ax = b$, where the matrix $A = [a_{ij}]_{3 \times 4}$ is given by

$$A = \begin{bmatrix} -1 & 5 & -1 & -3 \\ 4 & -1 & 2 & 6 \\ 3 & 4 & 1 & 3 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(a) Find all the possible vectors \mathbf{b} for which the linear system has non-trivial solution. (5%)

(b) Determine the solution \mathbf{x} . (5%)

4. Use Green's theorem to evaluate $\oint_C \vec{F} \cdot d\vec{R}$, where $\vec{F} = xy\vec{i} + xy^2\vec{j}$ and C : the triangle with vertices $(0,0)$, $(3,0)$, $(0,5)$. Note that the curve C is oriented counterclockwise. (8%)

5. Let $\{v_1, v_2\}$ span the vector space of inner product in R^2 . Please answer the following questions.

(a) Is it true that v_1 and v_2 must be mutually orthogonal? Explain why or why not. (3%)

(b) Give two examples showing that $\{v_1, v_2\}$ is an orthonormal base in R^2 . (4%)

注意：背面有試題

參考用

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Fourier Analysis, Partial Differential Equation and Complex Analysis (25%)

6. The function

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$$

- (a) Expand $f(x)$ in a Fourier series. (5%)
(b) Expand $f(x)$ in a complex Fourier series. (5%)

7. (a) Solve the partial differential equation (5%)

$$\frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} = 0$$

(b) Solve the boundary-value problem (10%)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi, \quad t > 0$$

$$u(0, t) = 0, \quad u(\pi, t) = 0, \quad t > 0$$

$$u(x, 0) = \sin x, \quad 0 < x < \pi.$$

程式語言 (25 %)

8. 以下三行 C 語言程式碼，請解釋變數 A 的意義 (5 %)

```
double *A;  
int num=100;  
A=(double *)malloc(num*sizeof(double));
```

9. 擬使用 "struct" 自訂一資料結構，內有三個變數，P、Q 與 R，其中 P 為單精度實數，Q 為雙精度實數，R 為整數，請以 C 語言寫出此一資料結構。(8 %)

10. 變數 i 為整數，x 為實數，i 由 0 到 10，x 則為以下的函數：(12%)

(1) 若 $i < 5$ ，則 $x = i * i$

(2) 若 $i \geq 5$ ，則 $x = i * i * i$

請使用 C 語言，以迴圈方式寫出上述的程式碼。

參考用