國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 機械工程研究所 丁組 科目: 丁工程數學 共 2 頁 第 1 頁

1. Solve the following problems

(1) Let $A=(a_{ij})$ be an $n\times n$ matrix such that for each $i=1,..n$

we have

$$\sum_{j=1}^{n} a_{ij}=0$$

Show that 0 is an eigenvalue of A. (8%)

(2) A recursive formula is given as

$$r_{n+1}=4r_n-t_n,\quad t_{n+1}=2r_n+t_n$$

with the initial values, $r_0=100$ and $t_0=10$. Determine

$\lim_{n\to\infty}\frac{r_n}{t_n}=?$ (8%)

(3) A nonhomogeneous system of equations is given as

$$x-2y+3z=1$$
$$2x+ky+6z=6$$
$$-x+3y+(k-3)=0$$

Determine the value of k for which (a) the system has a unique solution (3%), (b) the system has no solution (3%), and (c) the system has general solution (3%).

參考用

2. Solve the following partial differential equation:

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}+1$$
$$\theta(x,0)=0$$
$$\frac{\partial\theta(0,t)}{\partial x}=0$$
$$\theta(L,t)=0$$

Please solve the problem **according to the following the steps:**

(1) Separation of variables: $\theta(x,t)=\psi(x,t)+\varphi(x)$. (5%)

(2) Solve $\varphi(x)$. (5%)

(3) The variable $\psi(x,t)$ needs a further separation of variable. (5%)

(4) Solve $\psi(x,t)$ and get the full answer of $\theta(x,t)$. (10%)

3. (a) (10%) Find the eigenvalues and eigenvectors of A, where A is defined as follows.

$$\mathrm{A}=\begin{pmatrix}-6&-4&-2\\-4&-6&-2\\-2&-2&-17\end{pmatrix}$$

(b) (5%) Find a diagonal matrix D and an orthogonal matrix P, such that

$$\mathrm{D}=\mathrm{P}^T\mathrm{AP}$$

(c) (10%) Find the solution $\mathrm{x}(t)$ for a linear system,

$$\frac{d\mathrm{x}(t)}{dt}=\mathrm{A}\mathrm{x}(t)$$

with its initial condition $\mathrm{x}(0)=(2,4,3)^T$.

注：背面有試題

4. In a computer language of your choice from Basic, Fortran, C, Pascal and LISP

(a) Write a computer program with comments to solve a root of $f(x)=x^3-7x$ using the Newton-Raphson Method. 10%

(b) Write a computer program with comments to solve the numerical solution of following first order initial value problem. 15%

$$y'=-ty^2, \quad y(2)=1, \quad 2 \leqq t \leqq 10$$