1. The stream function for flow around a circular cylinder with a radius, a, can be expressed as

$$\psi = Ur(1-\frac{a^2}{r^2})\sin\theta$$

(a) Show that the velocity field represented by this stream function satisfies the continuity equation. (5%)

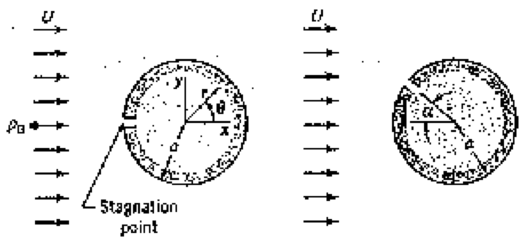
(b) Is this an irrotational flow field? If it is an irrotational flow, find the corresponding velocity potential. (5%)

(c) Determine the expression for the pressure coefficient,

$$C_p = \frac{p_s - p_\infty}{\frac{1}{2}\rho U^2}$$

where p_s is the surface pressure of the cylinder. (5%)

(d) If a small hole is located at the stagnation point, the stagnation pressure, p_{stag}, can be measured and used to determine the approach velocity, U. Show how p_{stag} and U are related. If the cylinder is misaligned by an angle, α, but the measure pressure still interpreted as the stagnation pressure, determine an expression for the ratio of the true velocity, U, to the predicted velocity, U'. (10%)



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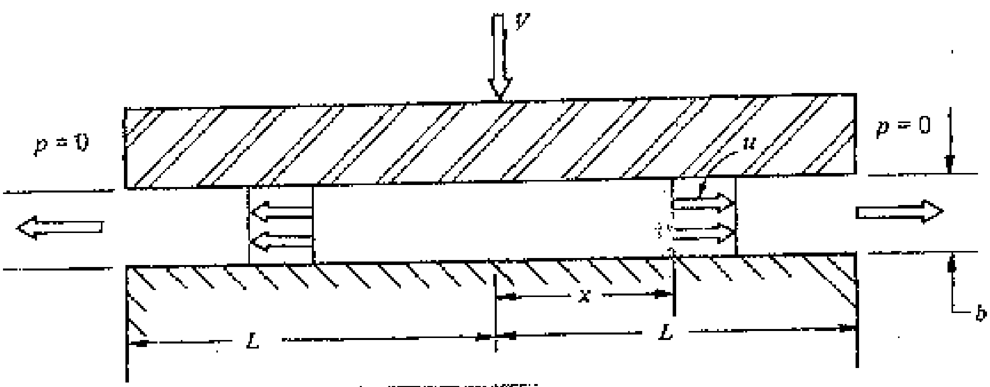
(2) Two very long parallel plates of length $2L$ are separated a distance b. The upper plate moves downward at a steady rate V. A nonviscous and incompressible fluid of density ρ fill the gap between the plates. Fluid is squeezed out between the plates, and since the flow is symmetrical, the velocity parallel to the plate at the center is zero. Assume that $b/L \to 0$ and that the velocity u parallel to the plate is substantially constant across the gap. Treat the flow as being one-dimensional and parallel to the x-axis. (25%)

(a) Show that the velocity at any point x from the center is approximately $u = Vx/b$. (10%)

(b) Noting that b changes with time and assume that the pressure outside the plate is zero, obtain an expression for the pressure at any point x along the plate. Neglect gravity. (15%)

Hint: Include an unsteady term in the Bernoulli equation:



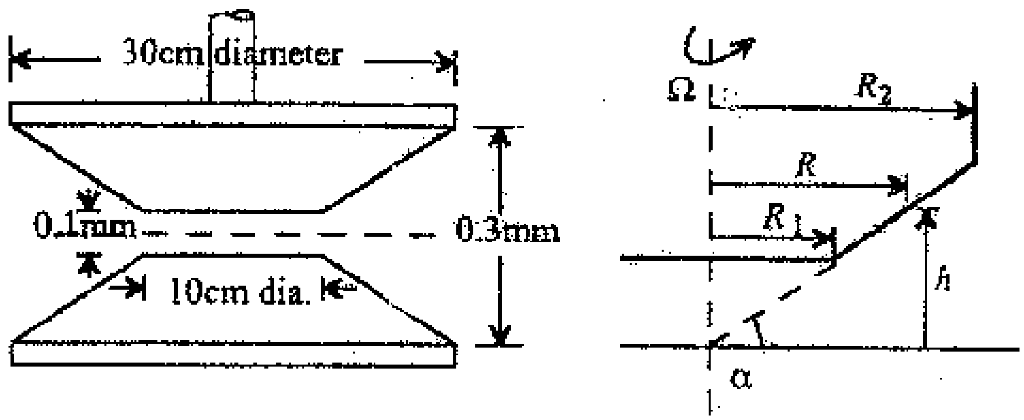
(3) A plate rotates coaxially with a stationary lower plate at an angular velocity of Ω=60 rpm . The space between the upper and lower plates is filled with oil (dynamic viscosity μ=0.1 Ns/m^2). The viscous torque is generated by the shear stress acting of the oil. (25%)

(a) Express the viscous force ($F_v = \int \tau dA$, τ=viscous stress) and the viscous torque (T_v= $F_v R$) on the inclined portion of the upper plate in terms of Ω, R_1, R_2, α , μ . (10%)

(b) Express the viscous forcee and the viscous torque on the flat portion of the upper plate in terms of Ω, R_1, R_2, α , μ . (10%)

(c) Compare the viscous torque between the inclined and central portion of the upper plate, and then discuss the effect of the inclined surface on the viscous torque. (5%)

α =inclined angle, R_1=radius of the flat portion of the upper plate. R_2=radius of the inclined portion of the upper plate.



(4) <u>Compressible flow</u> **(total 25 %)**

(a) Define the speed of sound (a) and the Mach number (M) (4 %)

(b) The energy equation for isentropic flow is $h + u^2/2 = h_o$. Show that:

$$a^2 + \frac{\gamma - 1}{2} u^2 = a_o^2 .$$ (4 %)

(c) What is f(M) in the relation between differential velocity changes and Mach number changes for isentropic flow of a perfect gas? That is: $\frac{du}{u} = \frac{dM}{M} f(M)$; and find f(M). (Hint: Differentiate $a^2 + \frac{\gamma - 1}{2} u^2 = a_o^2$ and replace $\frac{da}{a}$ by a combination of $\frac{du}{u}$ and $\frac{dM}{M}$ from logarithmic differentiation of the Mach number. (11 %)

(d) When $M \rightarrow 0$, f(M) = ? As $M \rightarrow \infty$, f(M) = ? Sketch f(M). (6 %)