1. A building is equipped with a Carnot heat pump which has a 2-kW power input. Measurements on the building indicate that the rate of heat gain will be 1.0 kW per Kelvin of temperature difference between the interior and exterior of the building. Find (a) the maximum permissible summer temperature, in degrees Celsius, that the heat pump (acting as an air conditioner) can handle if the interior is kept at 25 °C and (b) the COP (Coefficient of Performance) of the air conditioner when it is operating at the maximum value. The heat pump is then used in the winter to maintain the interior temperature at 20 °C. (c) For the same heat-transfer-rate across the walls and the same power input, determine the rate of heat transfer from the atmosphere to the heat pump, in kJ/min and (d) the COP, if the outside temperature is -10 °C. (25%)

2. Near the triple point on P-T phase diagram of a pure substance, the slope of melting line is generally steeper than that of vaporization line, i.e., $(dP/dT)_{sg} > (dP/dT)_{fg}$. (a) Give a thermodynamic explanation for this fact from appropriate equations of the phase transition. (b) If $dP/dT = 0.035$ atm/°C at the boiling point of water at 100 °C and 1 atm, estimate the latent heat of vaporization h_{fg} (in the dimension of cal/g) of water at 100 °C by using perfect-gas approximation. Note that the universal gas constant is 0.082056 atm-m^3/kgmole-K; 1 bar = 0.986923 atm; 1 cal = 4.1868 joule. (25%)

3. An electric current is used to heat a tube through which a suitable cooling fluid flow. The outside of the tube is covered with insulation to minimize heat loss to the surroundings, and thermocouples are attached to the outer surface of the tube to measure the temperature. Assuming uniform heat generation in the tube, derive an expression for the convection heat-transfer coefficient on the inside of the tube in terms of the measured variables: voltage E, current I, outside tube wall temperature T_o, inside and outside radii r_i and r_o, tube length L, and fluid temperature T_f. (15%)

Supplementary data:

(1) The conduction equation in a cylindrical coordinate is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\frac{\partial T}{\partial t}$$

4. Liquid metals have very low Prandtl numbers. This means that the diffusion of heat is
25% much more rapid than the diffusion of momentum. Taking this fact into account, you are asked to determine the Nusselt number for the flow of such a fluid over an isothermal flat plate by the following steps.

(a) Plot the velocity and thermal boundary layers for the flow over an isothermal flat plate. (5%)

(b) Write down the related governing equations and boundary, initial conditions. What assumptions you should make for simplying this equations? (5%)

(c) Transfer the governing partial differential equations to an ordinary differential equation by the method of similarity transformation. (5%)

(d) Solve this ODE to find out the temperature distribution of the fluid in the boundary layer. (5%)

(e) Also determine the local heat transfer coefficient and Nusselt number. (5%)

5. An enclosure consists of three infinitely long gray surfaces which form an equilateral
10% cross section (see figure below). Surface 1 has a known temperature T_1; surface 2 has a known net heat flux q_{2net} but an unknown temperature; surface 3 has a known temperature T_3. In the figure below, values for T_1, T_3, q_{2net}, ε_1, ε_2, and ε_3, are given.

(a) Draw the thermal circuit for the problem.

(b) Calculate q_{1net}, T_2 and q_{3net}.