

**Ordinary Differential Equations**

1. Find the solution for the following ordinary differential equations (ODEs):

(a)  $y' + y = -x/y$  (5%)

(b)  $y'' + 4y' + 3y = 65 \cos(2x)$  (5%)

(c)  $\begin{cases} y_1' = y_1 + y_2 + 10 \cos x \\ y_2' = 3y_1 - y_2 - 10 \sin x \end{cases}$ , find  $y_1$  and  $y_2 = ?$  (5%)

2. For a homogenous ODE given as:

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0 \quad (1)$$

(a) Find three solutions  $y_1(x)$ ,  $y_2(x)$ , and  $y_3(x)$  that can form a basis of solutions, show that they are linear independent, for Eq. (1). (5%)

(b) If there is a non-homogenous term  $r(x) = x^4 \ln x$  of Eq. (1), then Eq. (1) becomes

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x \quad (2)$$

find the particular solution for Eq. (2),  $y_p(x) = ?$  (5%)

**Laplace Transform and Fourier Analysis**

3. Solve the integro-differential equation  $y'(t) = 1 - e^{-2t} \int_0^t y(\tau) e^{2\tau} d\tau$ ,  $y(0) = 1$  (10%)

4.  $f(x) = x^2$ ,  $0 < x < 2\pi$ ,  $f(x) = f(x + 2\pi)$

(a) Find the Fourier series. (10%)

(b) Evaluate  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = ?$  (5%)

**Linear Algebra and Vector Analysis**

5. Solve the following linear systems

$$(a) \begin{bmatrix} 2 & 3 & 1 & -11 \\ 5 & -2 & 5 & -4 \\ 1 & -1 & 3 & -3 \\ 3 & 4 & -7 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \\ -7 \end{bmatrix} \quad (8\%) \quad (b) \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad (7\%)$$

參考用

6. Inscribed circles on the sheet metal surfaces are routinely used to investigate the forming limit diagram (FLD) of industrial stamping processes. The deformed sheet stretched from a point P:( $x_1, x_2$ ) to Q:( $y_1, y_2$ ) can be experimentally measured in a specific direction (principal direction) such that the eigenvalue problems can be applied. The solution procedure typically starts from a boundary circle  $x_1^2 + x_2^2 = 1$  and stretches to

$$\frac{y_1^2}{\lambda_1^2} + \frac{y_2^2}{\lambda_2^2} = 1, \text{ where } \lambda_1, \lambda_2 \text{ are eigenvalues. The transformation matrix A is designated}$$

$$\text{as } y = Ax; \text{ or } \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 1.5 \\ 1.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ can be determined experimentally. Please solve}$$

eigenvalues and eigenvectors to find the principal directions and indicate the shape of the deformed boundary. (10%)

注意：背面有試題

程式設計

7. (15%) Heron's formula for the area  $A$ , of a triangle with sides of length  $a, b, c$  is

$$A = \sqrt{[s(s-a)(s-b)(s-c)]},$$

where

$$s = \frac{(a+b+c)}{2}.$$

Write a function that accepts the value of  $a, b$ , and  $c$  as parameters from a calling function, and then calculates the values of  $[s(s-a)(s-b)(s-c)]$ . If this quantity is positive, the function calculate  $A$ . If the quantity is negative,  $a, b$ , and  $c$  do not form a triangle, and the function should set  $A = -1$ . The value of  $A$  should be returned by the function. The code is limit to C, C++, Visual Basic or Fortran programming language, and please state before your answer. All variables are declared to real numbers.

8. (10%) In sorting techniques, the most familiar algorithm is the bubble sort, in which successive value in the array are compared, beginning with the first two element. If the array is to be sorted in ascending (from smallest to largest) order, the smaller value of the two being compared is always placed before the larger value. The pseudo-code is as follows:

*For the first element in the array to 1 less than the last element (i index)*

*For the second element in the array to the last element (j index)*

*If num[j] < num[j-1] then*

*Swap num[j] with num[j-1]*

*End if*

*End For*

*End For*

Write a program to implement the bubble sort. The code is limit to C, C++, Visual Basic or Fortran programming language, and please state before your answer. All variables are declared to real numbers except for  $i, j$  index to the integer.

參考用

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