所別：機械工程學系碩士班 丁組(系統)(一般生)　　科目：工程數學(含程式設計) 共 2 頁 第 1 頁
機械工程學系光機電工程碩士班 甲組(機電系統控制)(一般生) 科目：工程數學及程式設計
本科考試可使用計算器，廠牌、功能不拘　　＊請在試卷答案卷（卡）內作答

參考用

Vector analysis and linear algebra

1. An elastic membrane stretched in a specific direction can be described as an eigenvalue problem. Please consider a membrane with a boundary circle $x_1^2 + y_1^2 = 1$ is stretched from a point P:(x_1,y_1) to Q:(x_2,y_2) and experimentally determined by

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

(a) Please find the principal directions by solving eigenvalues and eigenvectors. **(10%)**

(b) Please show that the deformed boundary is an ellipse by using $\frac{x_2^2}{\lambda_1^2} + \frac{y_2^2}{\lambda_2^2} = 1$, where

$\lambda_1 \lambda_2$ are eigenvalues **(5%)**

2. Please use Gauss elimination method to solve the following linear systems of electrical networks

(a) $\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 0 \\ 20 & 10 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 90 \\ 80 \end{bmatrix}$ **(5%)**

(b) $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$ **(5%)**

Ordinary differential equations

3. Find a general solution to the following ordinary differential equations (ODEs):

(a) $y' + xy = xy^{-1},\ y(0) = -\frac{1}{3}$ **(5%)**

(b) $y'' - 2y' + y = 70x^{3/2}e^x$ **(5%)**

4. For a homogenous ODE given as $y'''+2y''-y'-2y=0$. (1)

(a) Find three solutions $y_1(x)$, $y_2(x)$, and $y_3(x)$ that can form a basis of solutions, show that they are linear independent, for Eq. (1). **(5%)**

(b) If there is a non-homogenous term $r(x)=1-4x^3$ of Eq. (1), then Eq. (1) becomes

$y'''+2y''-y'-2y=1-4x^3$, (2)

find the particular solution for Eq. (2), that is $y_p(x)$ = ? **(5%)**

(c) Transfer Eq. (2) to a system of 1st-order ODEs and express the 1st-order system of ODEs in a matrix form. **(5%)**

注意:背面有試題

參考用

Partial differential equations and complex analysis

5. Solve

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ 0<x<1,\ t>0,$$

$$u(0,t)=u_0(t),\ u(1,t)=u_1(t),\ t>0,$$

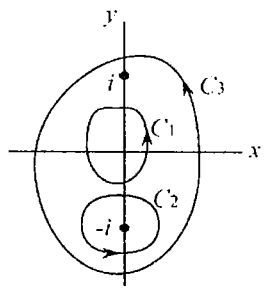
$$u(x,0)=h(x),\ 0<x<1.$$

(a) For $u_0(t)=u_1(t)=0$, $h(x)=\sin \pi x$. **(5%)**

(b) For $u_0(t)=u_1(t)=0$, $h(x)=x$. **(5%)**

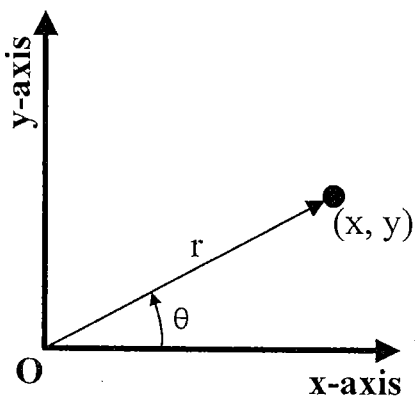
(c) For $u_0(t)=0$, $u_1(t)=\exp(-t)$, $h(x)=x$. **(5%)**

6. Evaluate the complex integral $\oint_C \frac{\sin z}{z^2+1}dz$ along the indicated closed contour, C_1, C_2 and C_3, respectively. **(10%)**



程式設計

7. **(10%)** 試寫一程式將卡氏座標系統(Cartesian coordinate system)中的一點(x, y) 轉換成極座標(polar coordinate)形式(r, θ)。在程式中 x 與 y 宣告為定值(x = 5.0, y = 5.0)即可，但若更動 x 與 y 值，程式編譯後需可計算出新的極座標值。程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，所有變數均以實數宣告，並註明使用的程式語言。



8. **(15%)** 有四個矩陣 A、B、C 及 D，維數分別為 M×N、N×M、M×M 及 M×M，若矩陣 A 、B 與 C 為已知，並且 D＝A × B ＋C，試寫一程式計算矩陣 D。在程式中 M 與 N 宣告為常數值(M = 4, N = 3)即可，但若更動 M 與 N 值，程式編譯後需可計算出新的矩陣 D。本程式中需使用迴圈計算，程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，除迴圈變數以整數宣告外，其餘所有變數均以實數宣告，矩陣內之數值無需考慮，並請註明使用的程式語言。

注意：背面有試題