

Notes: The statistical tables you may use in this exam are listed in page 2.

**Problem 1 (10%)** Let  $X$  be a random variable with probability distribution

$$f(x) = \begin{cases} (1+2x)/2, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability distribution of the random variable  $Y = X^2$ .

**Problem 2 (20%)** The answering machines of manufacturer A have a mean lifetime of 7 years and a standard deviation of 0.9 year, while those of manufacturer B have a mean of 6.5 years and a standard deviation of 0.8 year. What is the probability that a random sample of 36 answering machines from manufacturer A will have a mean lifetime that is at least 1 year more than the mean lifetime of a sample of 49 answering machines from manufacturer B?

**Problem 3 (20%)** A study is made to determine if a cold climate results in more students being absent from school during a semester than a warmer climate. Two groups of students are selected at random, one group from a cold area A and the other group from a warmer area B. Of the 300 students from A, 60 were absent at least 1 day during the semester, and of the 400 students from B, 52 were absent 1 or more days. Find a 95% confidence interval for the difference between the fractions of the students who are absent in the two areas.

**Problem 4 (20%)** The average length of time for students to register for fall classes at a certain college has been 60 minutes with a standard deviation of 10 minutes. A new registration procedure using modern computing machines is being tried. If a random sample of 12 students had an average registration time of 52 minutes with a standard deviation of 11 minutes under the new system, test the hypothesis that the population mean is now less than 60, using a level of significance of (1) 0.05, and (2) 0.01. Assume the population of times to be normal.

**Problem 5 (15%)** Three cards are drawn from an ordinary deck of playing cards, with replacement, and the number  $Y$  of spades is recorded. After repeating the experiment 64 times, the following outcomes were recorded:

y	0	1	2	3
f	21	31	12	0

Test the hypothesis at the 0.05 level of significance that the recorded data may be fitted by the binomial distribution  $b(y; 3, 1/4)$ , ie,  $p(y) = C_y^3 (1/4)^y (3/4)^{3-y}$ , for  $y = 0, 1, 2, 3$ .

**Problem 6 (15%)** Apply the least square estimation to Estimate a nonlinear regression equation  $y = b^x$ , for the following data:

x	1	2	2	3	5	6
y	7	20	33	90	1090	2900

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Table IV  
Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5518	0.5558	0.5598	0.5638	0.5677	0.5717	0.5757
0.2	0.5797	0.5837	0.5877	0.5917	0.5957	0.5997	0.6037	0.6077	0.6117	0.6157
0.3	0.6197	0.6237	0.6277	0.6317	0.6357	0.6397	0.6437	0.6477	0.6517	0.6557
0.4	0.6597	0.6637	0.6677	0.6717	0.6757	0.6797	0.6837	0.6877	0.6917	0.6957
0.5	0.6997	0.7037	0.7077	0.7117	0.7157	0.7197	0.7237	0.7277	0.7317	0.7357
0.6	0.7397	0.7437	0.7477	0.7517	0.7557	0.7597	0.7637	0.7677	0.7717	0.7757
0.7	0.7797	0.7837	0.7877	0.7917	0.7957	0.7997	0.8037	0.8077	0.8117	0.8157
0.8	0.8197	0.8237	0.8277	0.8317	0.8357	0.8397	0.8437	0.8477	0.8517	0.8557
0.9	0.8597	0.8637	0.8677	0.8717	0.8757	0.8797	0.8837	0.8877	0.8917	0.8957
1.0	0.8997	0.9037	0.9077	0.9117	0.9157	0.9197	0.9237	0.9277	0.9317	0.9357
1.1	0.9397	0.9437	0.9477	0.9517	0.9557	0.9597	0.9637	0.9677	0.9717	0.9757
1.2	0.9797	0.9837	0.9877	0.9917	0.9957	0.9997	1.0000	1.0000	1.0000	1.0000

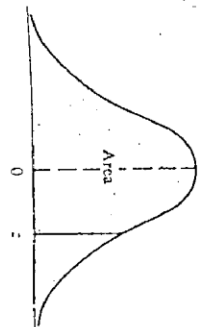


Table V†  
Critical Values of the t Distribution

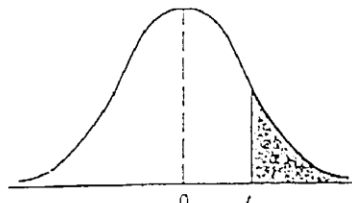
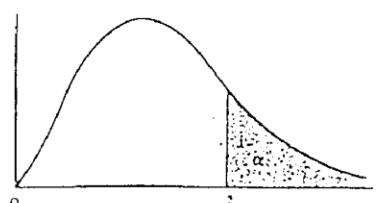


Table VI† Critical Values  
of the Chi-Square Distribution



v	alpha				
	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576

v	alpha							
	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	0.0+393	0.0+0201	0.0+982	0.0+393	3.841	5.024	6.635	7.879
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597
3	0.0717	0.115	0.216	0.352	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.483	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.932	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.633	44.181
24	9.886	10.856	12.461	13.848	36.415	39.364	42.980	45.558
25	10.520	11.524	13.220	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.944	15.379	38.885	41.923	45.642	48.290
27	11.803	12.879	14.673	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.408	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.150	17.708	42.557	45.722	49.588	52.335
30	13.787	14.953	16.891	18.493	43.773	46.979	50.892	53.672