

參考用

$$1) \frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + 6y = \delta\left(\frac{t}{2} - 1\right), \quad \delta(t) \text{ 代表 Delta}$$

函數, $y(0) = y'(0) = 0$, 請用 Laplace 轉換方法求出 $y(t)$. (25%)

$$2) x^2 y'' + x y' + (x^2 - 9)y = 0, \quad \text{設 Frobenius series 為 } y(x) = x^a \sum_{n=0}^{\infty} x^n, \quad \text{請算出 } a \text{ 的值. (25\%)}$$

$$3) \frac{d^4 y}{dt^4} + 2 \frac{d^3 y}{dt^3} - 3 \frac{d^2 y}{dt^2} - 8 \frac{dy}{dt} - 4y = 0, \quad \text{請算出 } y(t) \text{ 的 general solution. (25\%)}$$

$$4) \text{ 令 } \vec{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad \vec{u}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \vec{f} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & -2 \\ -\frac{1}{2} & 0 \end{bmatrix},$$

$$\frac{d\vec{u}}{dt} = A\vec{u} + \vec{f}, \quad \text{求 } u_1(t) \text{ 在 } t = \pi \text{ 時的}$$

值 $u_1(\pi)$. (25%)