國立中央大學101學年度碩士班考試入學試題卷

所別：土木工程學系碩士班 結構組(一般生)　科目：工程數學　共 1 頁　第 1 頁

本科考試可使用計算器，廠牌、功能不拘

*請在試卷答案卷（卡）內作答

1) Let $F(s)=\dfrac{e^{-3s}}{s^2-s+1}$ be the Laplace transform of a function $f(t)$. Please compute the value of $f(t)$ at the time $t=5$. (30 points)

2) Let $M=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$ and $K=\begin{bmatrix}2 & -2\\ -2 & 6\end{bmatrix}$ be two matrices. The vector $\vec{u}(t)=\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}$ satisfies the differential equation system $M\dfrac{d^2\vec{u}}{dt^2}+K\vec{u}=0$. The initial values of $\vec{u}(t)$ are $\vec{u}(0)=\begin{bmatrix}2\\ -2\end{bmatrix}$ and $\dfrac{d\vec{u}}{dt}(0)=\begin{bmatrix}0\\ 0\end{bmatrix}$. Please compute $\vec{u}(\dfrac{\pi}{2})$, which is the value of $\vec{u}(t)$ at the time $t=\dfrac{\pi}{2}$. (30 points)

3) The coordinates of three points A, B, and C are $A:(x,y,z)=(3,0,0)$, $B:(x,y,z)=(0,2,0)$ and $C:(x,y,z)=(0,0,4)$. The line AB connects A and B, the line BC connects B and C, and the line CA connects C and A. A triangular contour Γ is formed by these three lines. Please evaluate the contour integral $\oint_\Gamma \vec{F}\cdot d\vec{r}=\oint_\Gamma(\tan x)dx+(\ln y)dy+(\cos z)dz$. You might carry out the integration along Γ from A to B, and then from B to C, and finally from C to A. (20 points)

4) The matrix $A=\begin{bmatrix}5 & -3\\ -1 & 2\end{bmatrix}$ can be expressed as $A=B+G$. Here B is a symmetric matrix and G is an anti-symmetric matrix. That is, $B^T=B$ and $G^T=-G$. Please compute $\det(e^A)$, the determinant of the matrix e^A. (20 points)