參考用

1. To solve a set of linear simultaneous equations numerically with an initial guess x_1^0, x_2^0 and x_3^0, two different iterative procedure were used.
The original equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

(5%)

iterative procedure 1

$$\begin{cases} x_1^{i+1} = \dfrac{1}{a_{11}}\left(b_1 - a_{12}x_2^i - a_{13}x_3^i\right) \\ x_2^{i+1} = \dfrac{1}{a_{22}}\left(b_2 - a_{21}x_1^i - a_{23}x_3^i\right) \\ x_3^{i+1} = \dfrac{1}{a_{33}}\left(b_3 - a_{31}x_1^i - a_{32}x_2^i\right) \end{cases}$$

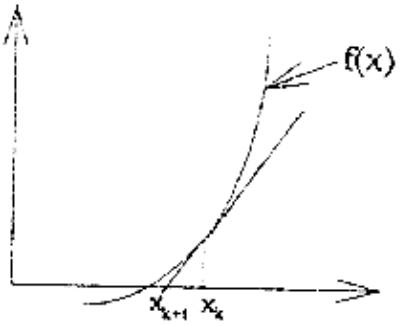
iterative procedure 2

$$\begin{cases} x_1^{i+1} = \dfrac{1}{a_{11}}\left(b_1 - a_{12}x_2^i - a_{13}x_3^i\right) \\ x_2^{i+1} = \dfrac{1}{a_{22}}\left(b_2 - a_{21}x_1^{i+1} - a_{23}x_3^i\right) \\ x_3^{i+1} = \dfrac{1}{a_{33}}\left(b_3 - a_{31}x_1^{i+1} - a_{32}x_2^{i+1}\right) \end{cases}$$

where the superscript i and i+1 are the iterative number.
Please explain which procedure needs less computing time and less storage memory to obtain the convergent solution.

2. From the following diagram please derive the mathematical relationship for solving $f(x) = 0$

$x_{k+1} \cong x_k - \dfrac{f(x_k)}{f'(x_k)}$ from

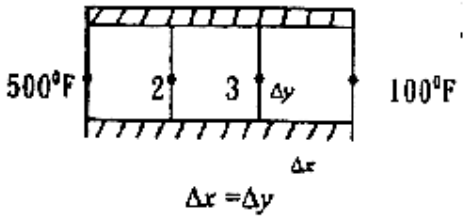


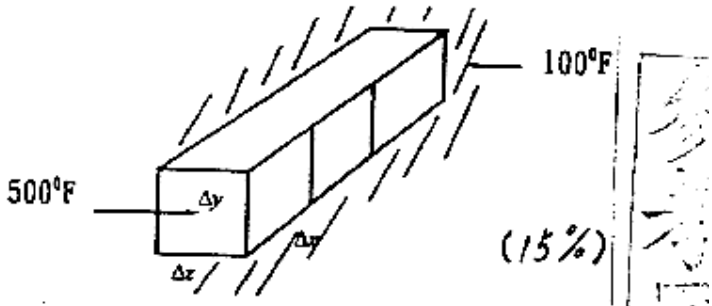
k: iteration number

(10%)

and solve $x = \cos x$
beginning with $x_0 = \pi/4$ (3 steps)

3. A solid rod conducts heat source at 500 °F to a heat sink at 100 °F. The sides of the rod are insulated as shown in the following figure





(15%)

i) Will this rod reach steady state temperature distribution finally ?

ii) Using the initial temperature distribution (step 0), please calculate the temperature distribution to step 2, through heat transfer equation. (R=40 $BTU/hr\text{-}ft\text{-}°F$)

	point			
step	1	2	3	4
0	500	300	200	100
1				
2				

4. A liquid consecutive unimolecular tpye first-order reactions such as $A \xrightarrow{k_1} R \xrightarrow{k_2} S$ in a bath reactor is started with a concentration of C_{A_0} of A, no R and S present. Find the concentrations of the components at all time. (15%)

5. Let tank A contains 10 gallons of brine in which 10 lbs of salt is dissolved and tank B contain 20 gallons of water.suppose water flows into tank A at the rate of 2 gallons per minute, and the mixture flows from tank A into tank B at 4 gallons per minute. From B 2 gallons are pumped back each minute to A while 2 gallons are flushed away.
   (a) Find the amount of salt in both tanks at all time. (15%).
   (You are asked to use Laplace Transform method.)
   (B) At what time is the amount of salt in tank B a maximum ?
   (5%)

6. (a) Prove that $\mathbf{F}=(2xz^3+6y)\mathbf{i} + (6x-2yz)\mathbf{j} + (3x^2z^2-y^2)\mathbf{k}$ is a conservative force field. (5%)
   (b) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is any path from (1,-1,1) to (2,1,-1). (10%)
   (c) Give a physical interpretation of the results. (5%)

7. Solve the PDE problem
   $u_t = \alpha^2 u_{xx}$ $0 < x < L$, $0 < t < \infty$
   with boundary conditions
   $u(0,t) = k_1$
   $u(L,t) = k_2$ $0 < t < \infty$
   and initial condition
   $u(x, 0) = \phi(x)$ $0 \le x \le L$ (15%)