所別：**網路學習科技研究所碩士班**　**不分組** 科目：**離散數學**

1. (15%) Determine the number of integer solutions of the equation
   $x_1+x_2+x_3+x_4=7$　where $x_i \geq 0$ for $i=1 \sim 4$

2. (15%) Prove that for all $n \geq 14$, n can be written as a sum of 3's and/or 8's.

3. （20%）Explain why, if there exists an NP-complete problem that can be solved in polynomial time, then all NP-complete problems can also be solved in polynomial time. Also explain why, if there exists an NP-complete problem whose lower bound is exponential time, then all NP-complete problems cannot be solved in polynomial time. Explain the importance of the two properties.

4. (20%) Analyze the best time complexity, the average time complexity and the worst time complexity of the binary search algorithm. (Given a value K and an ordered list of data, the binary search algorithm can find the position in the list where the value K occurs.)

5. (15%) Please show that it is impossible to have a finite state machine to represent $A=\{0^i 1^i \mid i \in Z^+\}$.

6. (15%) Solve the recurrence relation $a_n+a_{n-1}-6a_{n-2}=0$, $n \geq 2$, and $a_0=-1$ and $a_1=8$.