國立中央大學99學年度碩士班考試入學試題卷

所別：通訊工程學系碩士班 乙組(通訊網路)(一般生)　科目：工程數學　共 2 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

以下共分 A、B 和 C 三部份，每一部份 50 分，任選兩部份做答。請在答案卷最前面先註明您選答那兩部份，未註明者，不得對改卷所挑選之部份有異議。

Part A 機率 (50 分)

1. (15%) Let X be the random variable of the duration in minutes of a telephone conversation. The probability density function (PDF) of X is modeled as

$$f_X(x)=\begin{cases}\dfrac{e^{-x/3}}{3} & x\ge 0,\\ 0 & \text{otherwise.}\end{cases}$$

(1) (5%) Find the cumulative distribution function (CDF) $F_X(x)$.

(2) (5%) What is the probability that a telephone conversation will last between 3 and 6 minutes?

(3) (5%) For calls that last at least 3 minutes, what is the conditional PDF of the call duration?

2. (15%) Random variables X and Y have joint PDF

$$f_{X,Y}(x,y)=\begin{cases}cxy & 0\le x\le 1, 0\le y\le 1.\\ 0 & \text{otherwise.}\end{cases}$$

where c is a constant.

(1) (5%) Find c.

(2) (5%) What is Var[X]?

(3) (5%) What is the covariance of X and Y (Cov[X,Y])?

3. (10%) The random vector $\mathbf{X}=[X_1\ X_2\ X_3]^T$ has PDF

$$f_{\mathbf{X}}(\mathbf{x})=\begin{cases}6e^{-\mathbf{a}^T\mathbf{x}} & \mathbf{x}\ge 0\\ 0 & \text{otherwise.}\end{cases}\quad\text{where}\quad \mathbf{a}=[1\ 2\ 3]^T.$$

(1) (5%) What is the marginal PDF $f_{X_1}(x_1)$?

(2) (5%) Are the components of the random vector $\mathbf{X}$ independent? Please give the proof.

4. (10%) Random variables X and Y have the joint PDF

$$f_{X,Y}(x,y)=\begin{cases}6y & 0\le y\le x\le 1\\ 0 & \text{otherwise.}\end{cases}$$

(1) (5%) What is the CDF of the random variable X?

(2) (5%) What is P[Y>(X/2)]?

參考用

注意：背面有試題

Part B 離散數學 (50 分)

1 (5%) Find the coefficient of $x^4y^2z^6$ in $(x+y+z)^{12}$.

2 (5%) Please find out how many zeros at the end of 100! .
[For example, there are two zeros at the end of 105500.]

3 (10%) Consider the polynomial $f(x)=4x^4+3x-9x^2-6x+2$, which clearly has integer coefficients. Please find out all of *irrational* roots in this polynomial.

4 (10%) Find all solutions to the system of congruences.
$x \equiv 2 \pmod 3$
$x \equiv 1 \pmod 5$
$x \equiv 3 \pmod 7$

5 (10%) Please prove $O(n^2) \subset O(5^n)$.
[Note that the symbol 'O' indicates "Big-O."]

6 (10%) Solve $a_n=3a_{n-1}-4$, $a_0=5$, using generating functions (GF).
[Note: please use GF to solve this question; otherwise, no score will be given.]
(a) (5%) After applying GF to this relation, what is the equation of G(x)?
(b) (5%) Solving for G(x), what is the a_n?

Part C 線性代數 (50 分)

1. (10%) Let W_1 and W_2 be two subspaces of a vector space V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

2. (6%) Let V be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. Prove that if S_2 is linearly independent, then S_1 is linearly independent.

3. (10%) Let V and W be vector spaces and $T : V \to W$ be a linear transformation. Suppose $\beta = \{v_1, v_2, \cdots, v_n\}$ is a basis for V and T is one-to-one and onto. Prove that $T(\beta) = \{T(v_1), T(v_2), \cdots, T(v_n)\}$ is a basis for W.

4. (6%) Let V be an inner product space over R. Prove that for all $x, y \in V$, $4\langle x, y\rangle = \|x+y\|^2 - \|x-y\|^2$ where $\langle , \rangle$ denotes the inner product.

5. (6%) Let T be a linear operator on an inner product space V and T^* denote the adjoint of T. Let $U = T + T^*$. Prove that $U = U^*$.

6. (12%) Let T be a linear operator on a finite-dimensional inner product space V. Prove that if $\langle T(x), T(y)\rangle = \langle x, y\rangle$ for all $x, y \in V$, then $\|T(x)\| = \|x\|$ for all $x \in V$.

注：背面有試題