以下共分 A, B, 及 C 三部份，每一部份 50 分，任選兩部份做答。請在答案卷最前面先註明您選答那兩部份，未註明者，不得對改卷所挑選之部份有異議。

Part A 機率 (50 分)

1. (10%)

(1) (5%) The number of database queries processed by a computer in any 10-second interval is a Poisson random variable, K, with $\alpha = 5$ queries. What is the probability that at least two queries will be processed in a 2-second interval?

(2) (5%) If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2, compute the distribution of X+Y.

2. (10%) X and Y have the joint PDF $f_{X,Y}(x,y) = \begin{cases} \lambda\mu e^{-(\lambda x+\mu y)} & x \ge 0, y \ge 0, \\ 0 & \text{otherwise.} \end{cases}$

Find the PDF of $W=Y/X$.

3. (10%) A bin contains 3 different types of disposable flashlights. The probability that a type 1 flashlight will give over 100 hours of use is 0.7, with the corresponding probabilities for type 2 and type 3 flashlights being 0.4 and 0.3, respectively. Suppose that 20 percent of the flashlights in the bin are type 1, 30 percent are type 2, and 50 percent are type 3.

(1) (5%) What is the probability that a randomly chosen flashlight will give more than 100 hours of use?

(2) (5%) Given the flashlight lasted over 100 hours, what is the conditional probability that it was a type j flashlight, j=1, 2, 3?

4. (20%) X and Y are random variables with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 5x^2/2 & -1 \le x \le 1; 0 \le y \le x^2, \\ 0 & \text{otherwise.} \end{cases} \quad \text{Let } A = \{Y \le 1/4\}.$$

(1) (5%) What is the marginal PDF $f_X(x)$?

(2) (5%) What is the conditional PDF $f_{X,Y|A}(x,y)$?

(3) (5%) What is $f_{Y|A}(y)$?

(4) (5%) What is E[Y|A]?

參考用

注意：背面有試題

Part B 離散數學 (50 分)

1. (4%)

   Let $p(x,y)$ and $q(x,y)$ denote two open statements: $p(x,y) := x^2 \geq y$, and $q(x,y) := x+2 < y$. If the universe of each of x, y consists of all real numbers, you please determine the *true value* for each of the following open statements.

   (a) (1%) $p(-3,8) \wedge q(-2,-3)$

   (b) (1%) $p(1/2, 1/3) \vee \neg q(-2,-3)$

   (c) (1%) $p(2,2) \rightarrow q(1,1)$

   (d) (1%) $p(1,2) \leftrightarrow q(1,2)$

2. (4%)

   Let U be a given universal with A, B $\subseteq U$, $|A \cap B|=3$, $|A \cup B|=8$ and $|U|=12$. How many subsets C $\subseteq U$ satisfy $A \cap B \subseteq C \subseteq A \cup B$, and C contains an even number of elements?

3. (4%)

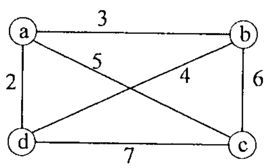
   Give a Big-O estimate for $f(n)=3n\log(n!)+(n+1)\log(n^2+1) + 3n^2$.

4. (4%)

   In how many different ways can eight identical cookies be distributed among three distinct children if each child receives at least two cookies and no more than four cookies.

5. (4%)

   Show all Hamilton circuits with minimum total weight.



參考用

6. (10%)

   Prove that the sum of the first n odd positive integers is n^2.

7. (10%)

   Please using "Generating Function (GF)" to solve the non-homogeneous recurrence relation $a_n = 3a_{n-2} + 1$, with initial condition $a_0 = a_1 = 1$.

   (Note: Please use GF to solve this question; otherwise, no score will be given.)

   (a) (5%) After applying GF to this relation, what is the equation of G(x)?

   (b) (5%) Solving for G(x), what is the a_n?

8. (10%)

   What are the solutions of the linear congruence: $3x \equiv 4 (\text{mod} 7)$.

   (a) (5%) Find an inverse of 3 modulo 7

   (b) (5%) What are the solutions of the linear congruence

注意：背面有試題

Part C 線性代數 (50 分)

1. Let $M_{n\times n}(R)$ represent the set of all $n \times n$ matrices with entries from the field of real numbers and W denote the subset of $M_{n\times n}(R)$ having trace equal to zero.
(a) (5%) Prove that W is a subspace of $M_{n\times n}(R)$.
(b) (4%) Find the dimension of W.
(c) (3%) Find a basis for W.

2. Let V and W be finite-dimensional vector spaces and $T : V \to W$ be a linear transformation from V to W.
(a) (4%) Prove that if $\dim(V)>\dim(W)$, then T cannot be one-to-one.
(b) (4%) Prove that if $\dim(V)<\dim(W)$, then T cannot be onto.

參考用

3. (8%) For the matrix $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 3 \\ 1 & 4 & 1 \end{pmatrix}$, compute the rank and the inverse if it exists.

4. (10%) Let V be an inner product space, and let $S = \{v_1, v_2, \cdots, v_n\}$ be an orthonormal subset of V. Prove that for any $x \in V$, we have $\|x\|^2 \geq \sum_{i=1}^{n} |\langle x, v_i\rangle|^2$ where $\langle , \rangle$ denotes the inner product.

5. (12%) Let T be a linear operator on an inner product space R^3. Define T by $T(a, b, c) = (5a + 4b - 2c, c - b, 5c)$. Test T for diagonalizability with an orthonormal basis, and find an orthonormal basis β such that $[T]_\beta$ is diagonal, if possible.