國立中央大學96學年度碩士班考試入學試題卷 共 2 頁 第 1 頁

## 所別:通訊工程學系碩士班 通訊系統與訊號處理組 科目:通訊系統

15% 1. In a communication system, two baseband signals are transmitted simultaneously by generating the RF signal

$$s(t) = m_1(t)\cos(\omega_c t) + m_2(t)\sin\omega_c t.$$

The carrier frequency is 20.25 MHz. The bandwidth of  $m_1(t)$  is 20 KHz and the bandwidth of  $m_2(t)$  is 10 kHz.

- (a) Evaluate the bandwidth of s(t).
- (b) Derive an equation for the spectrum of s(t) in terms of  $M_1(f)$  and  $M_2(f)$ , where  $M_1(f)$  and  $M_2(f)$  are spectra of  $m_1(t)$  and  $m_2(t)$ , respectively.
- 20% 2. Let *X(t)* and *Y(t)* be statistically independent Gaussian random processes, each with zero mean and unit variance at any time instant. Define the process:

$$Z(t) = X(t)\cos(2\pi t + \theta) + Y(t)\sin(2\pi t + \theta)$$

- (a) If  $\theta$  is a deterministic constant, determine the joint probability density function of the random variable  $Z(t_1)$  and  $Z(t_2)$  obtained by observing Z(t) at time instants  $t_1$  and  $t_2$ , respectively.
- (b) If  $\theta$  is a deterministic constant, is the process Z(t) stationary? Please explain for your answer.
- 15% 3. A phase modulation (PM) system uses a pair of pre-emphasis and de-emphasis filters defined by the transfer functions

$$H_{pe}(f) = 1 + j \frac{f}{f_0}$$
 and  $H_{e}(f) = \frac{1}{(1+j\frac{f}{f_0})}$ 

The power spectral density of the noise at the phase discriminator output is assumed to be constant and bandlimited  $(|f| \le W)$  in the absence of pre-emphasis and de-emphasis for the PM system. Show that the improvement in output signal-to-noise ratio produced by using this pair of filters is

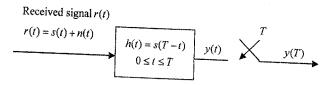
$$G = \frac{W/f_0}{\tan^{-1}(W/f_0)}$$

where W is the message bandwidth . (  $\int_{a^2+u^2}^{\frac{\partial u}{a^2+u^2}} = \frac{1}{a} \tan^{-1}(\frac{u}{a}) + C$  )

注:背面有試題

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10% 4. Please plot a correlator implementation of the following matched-filter receiver and show the output y(T) of the correlator receiver is equivalent to that of the matched-filter receiver.



20% 5. Consider the following set of three finite-energy signals

$$S_0(t) = 1 \qquad 0 \le t \le T$$
 
$$S_1(t) = \cos 2wt \qquad 0 \le t \le T, w = \frac{2\pi}{T}$$
 
$$S_2(t) = \sin^2 wt \qquad 0 \le t \le T$$

Please use the Gram-Schmidt procedure to obtain an orthonormal basis for the space spanned by these three signals.

20% 6. Consider a discrete memoryless channel (DMC) with input, output and transition probabilities given by p(x), p(y) and p(y|x) respectively, where  $x \in \{x_1, x_2, \dots x_N\}$  and  $y \in \{y_1, y_2, \dots y_M\}$ . The entropy functions for the DMC channel are defined as

$$H(X) = -\sum_{i=1}^{N} p(x_i) \log_2 p(x_i) \qquad H(Y) = -\sum_{j=1}^{M} p(y_j) \log_2 p(y_j)$$

$$H(Y|X) = -\sum_{i=1}^{N} \sum_{j=1}^{M} p(x_i, y_j) \log_2 p(y_j|x_i)$$

- (i) Please show that  $H(X) \le \log_2 N$  and  $H(Y) \le \log_2 M$ .
- (ii) Please show that  $H(Y|X) \le H(Y)$ .