所別：通訊工程學系碩士班 乙組(一般生) 科目：工程數學

注意: 本試題共分三部份 (每部份 50分) 請考生任選兩部分作答

PART I 線性代數 (50分)

1. (10%) Define the following terms:
   (a). Linear transformation. (5%)
   (b). Markov process. (5%)

2. (10%) Let $T:V\rightarrow W$ be a linear transformation. Prove that if T^{-1} exists, then T^{-1} is also a linear transformation.

3. (20%) Let the $A=\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$
   (a). Find the eigenvalues and corresponding eigenvectors for A. (10%)
   (b). Form the matrix A into a product SDS^{-1}, where D is diagonal. (5%)
   (c). Using (b) to calculate the A^5. (5%)

4. (10%) Let A be an $n\times n$ real symmetric matrix. Assume the eigenvalues of A are $\lambda 1, \lambda 2, \ldots, \lambda n$.
   (a). Find the eigenvalues of A. (5%)
   (b). If A is invertible, find the eigenvalues of $(AA^t)^{-1}$. (5%)

注意：背面有試題

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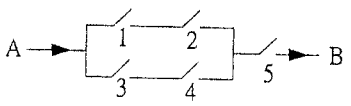
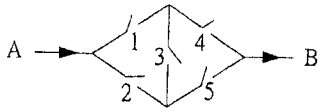
PART II 機率 (50分)

1. (20%) Relays are often used to control the current flows in circuits. Assume the probability of the closing of the i-th relay in the circuits is given by Pi, i=1,2,3,4,5. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits? (10% for each circuit)



(a)



(b)

2. (10%) Three players A, B and C simultaneously toss coins to determine who is the winner. The coin tossed by players A, B, and C turns up heads with probabilities P_A, P_B and P_C, respectively. If one person gets a different outcome from the other two, then he is the winner. If there is no winner, the players flip again and continue to do so until they get a winner. What is the probability that A will be the winner?

3. (10%) In a robust system, redundant components are allocated to increase the degree of robustness. For example, a satellite system consists of n components and functions on any given day if at least k of the n components function on that day. On a rainy day each of the components independently functions with probability P_{rainy}, whereas on a dry day they each independently function with probability P_{dry}. If the probability of rain tomorrow is P_x, what is the probability that the satellite system will function ?

4. (10%) A point is chosen at random on a line segment of length L. Find the probability that the ratio of the shorter segment to the longer segment is less than 1/4.

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PART III. 離散數學 (50分)

1. (20%) Let $X=\{x_1, x_2, ...,x_{20}\}$ be a set of 20 distinct integer numbers to be sorted by the Bubble sort technique.
   (a) After how many comparisons will the 10 smallest numbers of the original list be correctly arranged in ascending order? (10%)
   (b) Let a(n) denote the number of comparisons needed to sort n numbers. What is the recurrence relation between a(n) and a(n-1)? (10%)

```
[Reference Code]
Bubble Sort()
{
    int i, j, temp, A[SIZE];   /*assume array is indexed from 1 to SIZE */
    for (i=1; i<n; i++)
        for (j=n; j>i; j--)
            if (A[j] < A[j-1])
            {
                temp = A[j-1];
                A[j-1] = A[j];
                A[j] = temp;
            }
}
```

2. (10%) Let $G=(V,E)$ be a loop-free weighted connected undirected graph. Each edge $e \in E$ is assigned a positive real number $w(e)$. For $n \in Z^+$, let $S=\{e_1, e_2,..., e_n\}$ be a edge subset of E ($S \subseteq E$) that includes no cycle in G. Modify Kruskal's algorithm in order to obtain a spanning tree of G is minimal among all the spanning trees of G that include all the edges in $S=\{e_1, e_2,..., e_n\}$.

3. (10%) Let $G=(V,E)$ be an undirected graph. Design an algorithm to determine whether G is a connected graph.

4. (10%) Let P and Q respectively denote the input alphabet and output alphabet for finite state machine (FSM). Assume $P=Q=\{0,1\}$. Construct a state diagram for a finite state machine that reverses (from 0 to 1 or from 1 to 0) the symbols appearing the 4th, in the 8th, in the 12^{th}, ..., positions of an input string $x \in P^+$. Let y denote the output string from FSM. If x=0000, then y=0001; If x= 000111, then y= 000011; If x=000000111, then y=000100101.