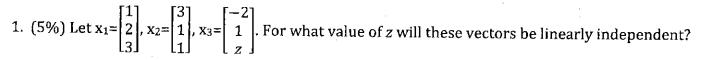
國立中央大學104學年度碩士班考試入學試題

所別:通訊工程學系碩士班 不分組(一般生) 科目:工程數學(機率、線性代數) 共 / 頁 第 / 頁 本科考試禁用計算器 *請在答案卷(卡)內作答





- 2. (5%) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ and $B = (I+A)^{-1}(I-A)$. Find the inverse matrix of (I+B).
- 3. (10%) Let A be an $m \times n$ matrix. Prove that rank(A)=n if and only if A^TA is an invertible $n \times n$ matrix.
- 4. (10%) Let T: V \rightarrow W be a linear transformation. Prove that the dim(V)=dim(Im(T))+dim(Ker(T)) is true, where dim(.), Im(.) and Ker(.) mean the dimension, image and kernel of a subspace (.) respectively.
- 5. (10%) Let A be an $n \times n$ matrix and λ be an eigenvalue of A.
 - (a) (3%) Clearly give the definition of "eigenspace."
 - (b) (7%) Prove $A-\lambda I$ is singular.
- 6. (10%) Use the matrix exponential to solve the initial value problem:

$$Y' = AY$$
, $Y(0) = Y_0$, where $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ and $Y_0 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

- 7.(8%) Box 1 contains 2 black and 8 white balls, and Box 2 contains 2 white and 8 black balls. Two balls are picked from a randomly selected box.
 - (a) (4%) Find the probability that both balls are black.
- (b) (4%) If both balls are white, find the probability that they came from Box 2.

8.(10%) The random variable X has C.D.F.
$$F_X(x) = \begin{cases} 0 & x < 1, \\ 0.2 & 1 \le x < 3, \\ 0.6 & 3 \le x < 4, \\ 0.9 & 4 \le x < 6, \\ 1 & x \ge 6. \end{cases}$$
 Define $Y = (X - 2)^2$.

- (a) (4%) Find and sketch C.D.F. $F_Y(y)$.
- (b) (6%) Find the expected value and the variance of Y.
- 9.(10%) Suppose the random variable X has p.d.f. $f_X(x) = kx$, $0 < x < 2\pi$, and $Y = \sin X$.
 - (a) (4%) Find the value of k.
 - (b) (6%) Find $f_Y(y)$.

10.(22%) The joint p.d.f. of random variables X and Y is
$$f_{XY}(x,y) = \begin{cases} \frac{3}{4}y & 0 < y < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) (6%) Find the p.d.f. of W = X Y.
- (b) (6%) Find the conditional p.d.f. of Y given X.
- (c) (10%) Find the correlation coefficient of X and Y.