所別：資訊工程學系碩士班　科目：資料結構與演算法
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1. Let T be a nonempty binary tree, leaves(T) be the number of leaves in T, and $n(T)$ be the number of nodes in T. Show that if leaves(t) = $\frac{n(T)+1}{2.0}$ then either both subtrees of T are empty or both subtrees of T are nonempty. (15%)

2. Show the result of inserting the following elements into an initially empty red-black tree: 28, 41, 22, 93, 11, 54. (10%)

3. A DNA sequence is composed of A, T, C, and G. A gene is a piece of DNA sequence. Please design a code table for the gene ATACCGA such that the code in the code table minimizes the length of the gene. (15%)

4. (a) Assume a group of 500 nodes might be declared as an array node (has two fields, i.e., info and next) as follows:

```
#define NUMNODES 500
struct nodetype   {
    int info, next;
 };
struct nodetype node[NUMNODES];
```

Please use loop (for example for (i =0; ...)) to write a program to place the nodes on the available list. Assume the global *variable* avail is used to point to the available list. The available list should be constructed as follows. The 500 nodes are initially linked, so that *node*[i] points to *node*[i+1]. *node*[0] is the first node on the available list, *node*[1] is the second, and so forth. *Node*[499] is the last node on the list, that is, *node*[499].*next* equals -1. (5%)

(b) Also please define a node (has two fields, i.e., info and next) using dynamic variables, that is, pointers. A node of this type is identical to the nodes of the array implementation in (a) except that the *next* field is a pointer rather than an integer. (5%)

5. A dominating set D of a graph $G = (V, E)$ is a subset of V such that every $v \in V$ is either in D or adjacent to at least one vertex of D. The dominating set decision (DSD) problem is defined as follows. Given an integer k and a graph $G = (V, E)$, does G have a dominating set of size $\leq k$? Write a nondeterministic polynomial time algorithm for the DSD problem. (8%)

6. A prune-and-search algorithm consists of several iterations. At each iteration, it prunes away a fraction of input data, and then invokes itself recursively to solve the problem for

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the remaining data. After a certain number of iterations, the size of input data will be so small that the problem can be solved directly in some constant time. Given a set of n integers, write a prune-and-search algorithm to find the kth smallest integer in $O(n)$ time complexity (8%). You should also prove that your algorithm is of $O(n)$ time complexity (9%).

7. Given an undirected graph $G = (V, E)$ with $n = |V|$ vertices, four vertices of G, say, u, v, x, and y, are said to form a 4-cycle if (u,v), (v,x), (x,y) and (y,u) are in E. Consider the problem of determining whether G contains a 4-cycle. A naïve method by checking all possible 4-combinations of the vertex set will need $\Omega(n^4)$ time to complete the job. Design a more efficient algorithm (i.e., the time complexity of your algorithm should be $O(n^k)$ with $k < 4$) to solve the problem (8%). Analysis the execution time of your algorithm (5%).

8. Consider the single-source shortest-paths problem: from a given vertex called the source in a weighted di-graph $G = (V, E)$, find shortest paths to all its other vertices. Dijkstra's algorithm is a famous algorithm for this problem. However, the algorithm is applicable to graphs with some specified condition only. The execution process of Dijkstra's algorithm can be decomposed into $|V| - 1$ stages. At each stage, the algorithm finds a shortest path from the source to a vertex. Describe such a process clearly on the following di-graph with vertex a as the source (8%). Under what condition Dijkstra's algorithm will not work? Give an example to explain your answer. (4%)