國立中央大學九十一學年度碩士班研究生入學試題卷

所別：資訊工程學系 不分組　　科目：線性代數　　共 1 頁 第 1 頁

※ 請務必按照題號次序寫在答案紙上。

1. (50%) 對錯申論題（一定要有說明或反例，每小題答對給 5 分，答錯扣 2 分，不答 0 分，本題總分≥ 0）
   (*a*) All elementary row operations are reversible.
   (*b*) The solution set of $Ax = b$ is obtained by translating the solution set of $Ax = 0$.
   (*c*) Let T be a linear transformation. If $\{v_1, v_2, v_3\}$ is linearly dependent, then $\{T(v_1), T(v_2), T(v_3)\}$ is also linearly dependent.
   (*d*) Let A be the square standard matrix of transformation T. T is one-to-one if and only if T is onto.
   (*e*) If A and B are $n\times n$ matrices, then $(A+B)(A-B) = A^2 - B^2$.
   (*f*) A plane in R^3 is a two-dimensional subspace of R^3.
   (*g*) A row replacement operation on matrix A doesn't change the determinant, and thus doesn't change the eigenvalues of A.
   (*h*) Similar matrices always have the same eigenvectors.
   (*i*) If P is an orthogonal vector set, then P is a linearly independent vector set.
   (*j*) If $W = \text{Span}\ \{v_1, v_2, \ldots, v_n\}$, then W and $W^\perp$ are always subspaces.

2. (10%) 是非題 (每小題答對給 2 分，答錯扣 2 分，不答 0 分，本題總分≥ 0)
   If A and B are two $n\times n$ matrices.
   (*a*) $\det A^{-1} = (\det A)^{-1}$.
   (*b*) $(A+B)^{-1} = A^{-1} + B^{-1}$.
   (*c*) $\det(A + B) = \det A + \det B$.
   (*d*) $(AB)^T = A^T B^T$.
   (*e*) $\det(ABC)^T = \det A^T \det B^T \det C^T$.

3. (10%) Let $B = \left\{\begin{bmatrix}7\\5\end{bmatrix},\begin{bmatrix}-3\\-1\end{bmatrix}\right\}$ and $C = \left\{\begin{bmatrix}1\\-5\end{bmatrix},\begin{bmatrix}-2\\2\end{bmatrix}\right\}$ be bases for R^2. Find the change-of-coordinates matrices from "B to C" and "C to B".

4. (10%) Find the bases for ***Col A***, ***Row A***, and ***Nul A***, where $A = \begin{bmatrix}1 & 3 & 3 & 2 & -9\\ -2 & -2 & 2 & -8 & 2\\ 2 & 3 & 0 & 7 & 1\\ 3 & 4 & -1 & 11 & -8\end{bmatrix}$.

參考用

5. (10%) Diagonalize matrix $A = \begin{bmatrix}4 & 2 & 2\\ 2 & 4 & 2\\ 2 & 2 & 4\end{bmatrix}$; that is to find matrices P and D such that $A = PDP^{-1}$.

6. (10%) Find a QR factorization of matrix $A = \begin{bmatrix}1 & 3 & 5\\ 1 & 1 & 0\\ 1 & 1 & 2\\ 1 & 3 & 3\end{bmatrix}$, where columns of Q form an orthonormal basis for Col A.