國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 資訊工程學系 不分組

※ 請務必按照題號次序寫在答案紙上。

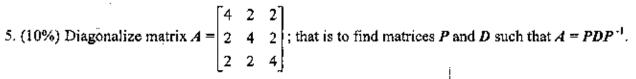
- 1. (50%) 對錯申論題 (一定要有說明或反例,每小題答對給 5 分,答錯扣 2 分,不答 0 分,本題總分≥ 0)
 - (a) All elementary row operations are reversible.
 - (b) The solution set of Ax = b is obtained by translating the solution set of Ax = 0.
 - (c) Let T be a linear transformation. If $\{\nu_1, \nu_2, \nu_3\}$ is linearly dependent, then $\{T(\nu_1), T(\nu_2), T(\nu_3)\}$ is also linearly dependent.
 - (d) Let A be the square standard matrix of transformation T. T is one-to-one if and only if T is onto.
 - (e) If A and B are $n \times n$ matrices, then $(A+B)(A-B) = A^2 B^2$.
 - (f) A plane in \mathbb{R}^3 is a two-dimensional subspace of \mathbb{R}^3 .
 - (g) A row replacement operation on matrix A doesn't change the determinant, and thus doesn't change the eigenvalues of A.
 - (h) Similar matrices always have the same eigenvectors.
 - (i) If P is an orthogonal vector set, then P is a linearly independent vector set,
 - (j) If $W = \text{Span } \{v_1, v_2, ..., v_n\}$, then W and W^{\perp} are always subspaces.
- 2.(10%) 是非題 (每小題答對給 2 分、答錯扣 2 分、不答 0 分、本題總分≥ 0)

If A and B are two $n \times n$ matrices.

- $(a) \det A^{-1} = (\det A)^{-1}.$
- $(b) (A+B)^{-1} = A^{-1} + B^{-1}$.
- (c) $\det(A + B) = \det A + \det B$.
- $(d) (AB)^T = A^T B^T.$
- (e) $\det(ABC)^T = \det A^T \det B^T \det C^T$.
- 3. (10%) Let $B = \left\{ \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix} \right\}$ and $C = \left\{ \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$ be bases for \mathbb{R}^2 . Find the change-of-coordinates

matrices from "B to C" and "C to B".

4. (10%) Find the bases for *Col A*, *Row A*, and *Nul A*, where $A = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$.



6. (10%) Find a QR factorization of matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$, where columns of Q form an orthonormal basis

for Col A.