※ 請務必按照題號次序寫在答案紙上。

1.(10%) Let A be an n by n matrix whose n column vectors are linearly independent. From the study of linear algebra, we know that the rank of A is n. Describe at least another 6 properties of A.

2.(10%) Determine the plane : $ax + by + cz = 1$ (i.e. find coefficients a, b, c) in R^3 that pass through points (-1, -2, 2), (2, 1, -1), and (3, -4, 2).

3.(20%) For each of the following two statements, if you think the statement is correct, then give a proof to prove that the statement is correct; otherwise give a counterexample to show that the statement is incorrect..

(a) If any three vectors v_1, v_2, v_3 in R^n are linearly independent, then the vectors $w_1 = v_1 + v_2$, $w_2 = v_1 + v_3$, $w_3 = v_2 + v_3$ are also linearly independent.

(b) If T is a linear transformation that maps R^n onto R^n (i.e. for any vector y in R^n, we can always find a vector x in R^n such that $T(x)= y$), then T must be one-to-one.

4.(5%) If A is a 5 by 5 matrix with $|a_{ij}| \le 1$, then $\det A \le$?. Give the upper bound as tight as possible and give a reason to justify your answer.

5.(5%) Compute the determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \\ 4 & 1 & 1 & 4 \\ -8 & -1 & 1 & 8 \end{vmatrix}$.

6.(20%) 對錯申論題（一定要有說明，每小題答對給 5 分，答錯扣 2 分，不答 0 分）

(a) Two vector sets $\left\{ \begin{bmatrix} a+3b \\ a-b \\ 2a-b \\ 4b \end{bmatrix} \middle| a,b \in R \right\}$ and $\left\{ \begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix} \middle| a,b,c \in R \right\}$ are both subspace of R^4.

(b) If A is an $m \times n$ matrix and Col $A = R^m$, then

(i) linear system $Ax = b$ has a unique solution for every b in R^m.

(ii) transformation $x \to Ax$ is one-to-one.

(c) If $n \times n$ matrix A has n linear-independent eigenvectors, then A is invertible and A^{-1} also has n linear-independent eigenvectors.

(d) If matrix A is diagonalizable, then the columns of A are linearly independent and A has n distinct eigenvalues.

7.(10%) Find a base for the orthogonal complement of column space of matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 4 & -6 & 8 \\ -2 & -3 & 2 \\ -4 & 1 & -3 \end{bmatrix}$.

8.(10%) Find a least-squares solution of $Ax = b$ and the least-squares error associated with the solution, where $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$.

9.(10%) Find a singular value decomposition of matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$.

Good Luck !