DISCRETE MATHEMATICS

1. Let $A = \{1,2,3,4,5\}\times\{1,2,3,4,5\}$, and define binary relation $\boldsymbol{R}$ on A by $(x_1, y_1)\boldsymbol{R}(x_2, y_2)$ if $x_1 + y_2 = x_2 + y_1$.
   (a) Verify that $\boldsymbol{R}$ is an equivalence relation on A.(9%)
   (b) Determine the equivalence classes $[(1,3)]$, $[(2,4)]$ and $[(1,1)]$. (6%)
   (c) Determine the partition of A induced by $\boldsymbol{R}$.(10%)

2. $K_{m,n,\ell}$ denotes a complete tripartite graph $G = (V, E)$ such that
   (1) $V = V_1 \cup V_2 \cup V_3$, where $|V_1| = m$, $|V_2| = n$, $|V_3| = \ell$, and $V_i \cap V_j = \varnothing$ if $i \neq j$.
   (2) There is an edge connecting vertices a and b if and only if $a \in V_i$, $b \in V_j$ and $i \neq j$.
   (a) Show that $K_{2,2,2}$ is planar.(8%)
   (b) Show that $K_{3,2,1}$ is nonplanar.(8%)
   (c) Find the necessary and sufficient condition in terms of m, n, and ℓ such that $K_{m,n,\ell}$ is planar.(9%)

3. Develop a general explicit formula for a nonhomogeneous recurrence relation of the form $a_n = ra_{n-1} + s$, where r, s and a_0 are given constants.
   (a) $r = 1$.(10%)
   (b) $r \neq 1$.(15%)

4. A fair die is tossed four times in succession. Find the probability that the four resulting numbers form a nondecreasing sequence.(25%)