國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 資訊工程研究所 不分組 科目: 線性代數 共 1 頁 第 1 頁

※ 請務必按照題號次序做答。

1. (50%) True and False.（一定要有說明、證明或反例；每小題 5 分）
   - (a) A consistent linear system has infinitely many solutions if and only if at least one column in the coefficient matrix does not contain a pivot position.
   - (b) The linear system $Ax = \mathbf{0}$ always has solution.
   - (c) The linear transformations of n linear-dependent vectors are linearly dependent.
   - (d) The geometric operations: translation, rotation, and scaling are linear transformation.
   - (e) If a linear transformation $T: R^n \to R^m$ is one-to-one, then $n = m$.
   - (f) A is invertible if and only if A^3 is invertible.
   - (g) $A^3 = \mathbf{0}$ if and only if $\det A = 0$.
   - (h) Let R^n be the set of all vectors with n entries. R^n is a subspace of R^{n+1}.
   - (i) If $n \times n$ matrix A has n linearly independent eigenvectors, then A is invertible.
   - (j) If matrix A has orthonormal columns, then $AA^T = \mathbf{I}$.

2. (10%) Calculate A^{21}, where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$.

3. (10%) Let A be an $n \times n$ invertible matrix. Show that $Ax = b$ has solution

$$x_i = \frac{\det A_i(b)}{\det A} \qquad i = 1, 2, \ldots, n$$

where $A_i(b)$ is the matrix obtained from A by replacing the ith column with vector b.

參考用

4. (10%) Find bases for Col A, Row A, Nul A, and Nul A^T, where $A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}$.

5. (10%) Show that if A is diagonalizable, then A^T and A^{-1} are also diagonalizable.

6. (10%) Find a QR factorization of matrix $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$.