國立中央大學八十五學年度碩士班研究生入學試題卷

所別：資訊工程研究所　不分組　科目：線性代數　共 1 頁 第 1 頁

※ 請務必按照題號次序做答。

1. (20%)Give the definitions of the following terms (每小題 4 分)
   (a) vector space and subspace.
   (b) linear independent and linear dependent vectors.
   (c) invertible matrix and elementary matrix.
   (d) one-to-one mapping and onto mapping.
   (e) similar matrix and diagonalizable matrix.

2. (40%)True and False. (一定要有說明、證明或反例；每小題 4 分)
   (a) A linear system with fewer equations than variables cannot have a unique solution.
   (b) Two linear systems $Ax = b$ and $Bx = c$ are equivalent if and only if A and B are row equivalent.
   (c) If a linear system has no free variables, then it has a unique solution.
   (d) A basis of a vector space is a maximal independent set and a minimal spanning set.
   (e) The subset of dependent vectors is dependent.
   (f) If AB is invertible, then B is invertible.
   (g) $V \cap V^{\perp}$ is always non-empty, where $V^{\perp}$ is the orthogonal complement of V.
   (h) If $n \times n$ matrix A has n linear-independent eigenvectors, then so do both A^{T} and A^{-1}.
   (i) If A is row equivalent to the identity matrix $\mathbf{I}$, then A is diagonalizable.
   (j) If A is diagonalizable, then the columns of A are linearly independent.

3. (10%)Give two geometric meanings for that the linear system $Ax = b$ is consistent.

4. (10%)Give two algorithms to find invertible matrix (you shall not use determinant).

5. (10%)Find bases for Col A, Row A, Nul A, and Nul A^{T}, where $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$.

6. (10%)Diagonalize the matrix A to PDP^{-1} and find P and D, where $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$.