類別：資工類　　　　科目：離散數學與線性代數　　共 三 頁　第 一 頁

本科考試禁用計算器　　　　＊請在試卷答案卷（卡）內作答

參考用

多重選擇題 (每題答對給 5 分、答錯每選項倒扣1分)

1. Let $[T_1]=\begin{bmatrix}1 & -2 & 2\\ 2 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$, $[T_2]=\begin{bmatrix}1 & -3 & 4\\ -1 & 1 & 1\\ 1 & -2 & 5\end{bmatrix}$, $[T_3]=\begin{bmatrix}1 & 4 & -3\\ 2 & 7 & 1\\ 1 & 1 & 3\end{bmatrix}$, and $[T_4]=\begin{bmatrix}1 & 4 & -3\\ 2 & 7 & 1\\ 0 & 1 & 3\end{bmatrix}$.

Which linear operators are one-to-one?
(A) T_1 (B) T_2 (C) T_3 (D) T_4 (E) none of the above.

2. Let $A=\begin{bmatrix}1/2 & -\sqrt{3}/2\\ 0 & 0\end{bmatrix}$, $B=\begin{bmatrix}\sqrt{2} & -\sqrt{2}\\ \sqrt{2} & \sqrt{2}\end{bmatrix}$, and $C=\begin{bmatrix}2 & 0\\ 0 & -2\end{bmatrix}$. Which of the following are true?

(A) Matrix A is the stand matrix for the composition of linear operators stated below: a counterclockwise rotation of 60°, followed by an orthogonal projection on the x-axis.
(B) Matrix B is the stand matrix for the composition of linear operators stated below: a dilation with factor k=2, followed by a counterclockwise rotation of 45°.
(C) Matrix C is the stand matrix for the linear operator defined below: a reflection about the x-axis, followed by a contraction with factor k=1/2.
(D) Matrix C is the stand matrix for the linear operator defined below: a reflection about the x-axis, followed by a dilation with factor k=2.
(E) None of the above.

3. Which of the following statements are false?
(A) If T: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}^m$ is a linear transformation and $m>n$, then T is one-to-one.
(B) If T: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}^n$ and $T(0)=0$, then T is linear.
(C) If T: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}^n$ is a linear operator and if $T(\mathbf{x})=3\mathbf{x}$ for some vector $\mathbf{x}$, then $\lambda=3$ is an eigenvalue of T.
(D) If T: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}^m$ and if $T(k\mathbf{x})=kT(\mathbf{x})$ for all scalars k and for all vectors $\mathbf{x}$ in $\boldsymbol{R}^n$, then T is linear.
(E) If T: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}^m$ and if $T(k\mathbf{x})=2kT(\mathbf{x})$ for all scalars k and for all vectors $\mathbf{x}$ in $\boldsymbol{R}^n$, then T is linear.

4. Which of the following sets of vectors in $\boldsymbol{R}^3$ are linearly independent?
(A) (3, 8, 7), (1, 5, 3), (2, −1, 2)
(B) (0, 1, 1), (3, 0, 0), (1, 0, −1)
(C) (1, −1, −2), (0, 0, −6), (4, 2, 2)
(D) (3, 0, −3), (0, 2, 3), (0, 1, 1).
(E) None of the above.

5. Let $\mathbf{u}=\begin{bmatrix}u_{11} & u_{12}\\ u_{21} & u_{22}\end{bmatrix}$ and $\mathbf{v}=\begin{bmatrix}v_{11} & v_{12}\\ v_{21} & v_{22}\end{bmatrix}$. With the standard matrix addition and the standard scalar multiplication, $\mathbf{u}+\mathbf{v}=\begin{bmatrix}u_{11}+v_{11} & u_{12}+v_{12}\\ u_{21}+v_{21} & u_{22}+v_{22}\end{bmatrix}$ and $k\mathbf{u}=\begin{bmatrix}ku_{11} & ku_{12}\\ ku_{21} & ku_{22}\end{bmatrix}$. Determine which of the following are subspaces of M_{22}.

(A) The set of all 2×2 matrices of the form $\begin{bmatrix}a & 2\\ 2 & b\end{bmatrix}$ with standard matrix addition and scalar multiplication.

(B) The set of all 2×2 matrices of the form $\begin{bmatrix}a & b\\ b & d\end{bmatrix}$ with standard matrix addition and scalar multiplication.

(C) The set of all 2×2 matrices of the form $\begin{bmatrix}a & a\\ c & d\end{bmatrix}$ with standard matrix addition and scalar multiplication.

(D) The set of all 2×2 matrices of the form $\begin{bmatrix}1 & 1\\ c & d\end{bmatrix}$ with standard matrix addition and scalar multiplication.

(E) None of the above.

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6. Which are the eigenvectors of matrix $\begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{bmatrix}$?

(A) $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$. (B) $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$. (C) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. (D) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. (E) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

7. Which are correct ?
(A) Any matrix has eigenvalues.
(B) An eigenvalue may be a complex number.
(C) A symmetric matrix always has real eigenvalues.
(D) A symmetric matrix always has positive eigenvalues.
(E) Matrix A^TA has no negative eigenvalue.

8. If A is a matrix and $W^\perp$ is the orthogonal complement of vector set W, then
(A) $W^\perp$ is always a subspace.
(B) $(W^\perp)^\perp = W$.
(C) $(\text{Col}\,A)^\perp = (\text{Row}\,A)$.
(D) $\text{Col}\,A = (\text{Nul}\,A)^\perp$.
(E) $(\text{Col}\,A)^\perp = \text{Nul}\,A^T$.

9. For linear system $Ax = b$, which are correct ?
(A) $Ax = b$ always has solution.
(B) If $Ax = b$ has no solution, then $Ax = b$ must has least-squares solution.
(C) Sometimes, A^TA is not invertible.
(D) Sometimes the least-squares solution $(A^TA)^{-1}A^Tb$ doesn't exist.
(E)) The least-squares solution can be set as $R^{-1}Q^Tb$.

10. If A is a matrix. Which are correct ? (A) A is invertible, thus $A = PDP^{-1}$. (B) A has linearly independent eigenvectors, thus $A = QR$. (C) A is symmetric, thus $A = PDP^T$. (D) A is symmetric, thus $A = \lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T + .. + \lambda_n u_n u_n^T$. (E) A is square, thus $A = U\Sigma V^T$ (*SVD*).

11. Let $P(x)$ be the statement "x = x^2". If the domain consists of the integers, which of the following propositions are true?

(A) $\forall x P(x)$ (B) $\forall x \neg P(x)$ (C) $\exists x P(x)$ (D) $\exists x \neg P(x)$ (E) $\exists! x P(x)$

12. Which of the following statements about RSA are correct?
(A) RSA is built on top of Fermat's little theorem.
(B) The two primes p and q chosen initially should be large enough.
(C) The encryption key e is co-prime to $(p-1)(q-1)$.
(D) The decryption key d is the inverse of e modulo $(p-1)(q-1)$.
(E) Both d and e will be made to the public. Hence RSA is also referred to as the public key system.

13. Let A be a set with n elements, and R be a relation on A. Which of the following statements about R are correct?
(A) The relation R can be represented by an n by n 0-1 matrix.
(B) The reflexive closure of R can be obtained quickly by adding some necessary 1's in the n by n matrix of R, but the symmetric closure of R can not.
(C) The transitive closure of R, R^*, is defined by the formula: $R^* = \bigcup_{i \in Z^+} R^i$
(D) In reality, the number of R^is needed to compute the transitive closure of R is less than n.
(E) The Roy-Warshall algorithm helps us quickly obtain the necessary R^is for the transitive closure of R.

參考用

14. Which of the following statements about Graph is correct?

(A) There are n nodes and $\frac{n(n-1)}{2}$ edges in K_n.

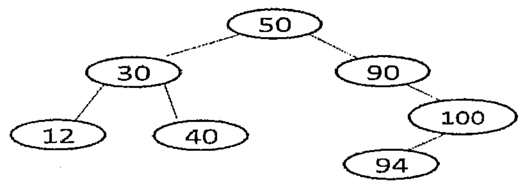
(B) There are n nodes and n edges in C_n, where $n \geq 3$.

(C) There are n nodes and $2n$ edges in W_n, where $n \geq 3$.

(D) There are 2^n nodes and $n*2^{n-1}$ edges in Q_n.

(E) There are $n+m$ nodes and $\frac{n*m}{2}$ edges in $K_{n,m}$.

15. Given the following Graph G. Which of the following statements are correct?



(A)G is a tree. **(B)**G is a binary search tree. **(C)**The length of the longest path in G is 3. **(D)**There are 7 connected components in G. **(E)**One possible DFS result of G (starting from node 50) is 50, 90, 100, 94, 30, 12, 40.

16. Suppose x and y are students, and we define the following predicates:

$c(x,y)$: x and y are classmates; $r(x,y)$: x and y are roommates; $d(x,y)$: x and y are in the same department;

Which of the following are logical equivalent to:

$\neg[\forall x \exists y, d(x,y) \wedge r(x,y) \rightarrow c(x,y)]$.

(A) $\exists x,y,[d(x,y) \wedge r(x,y) \rightarrow c(x,y)]$. (B) $\forall x,y,[c(x,y) \rightarrow d(x,y) \wedge r(x,y)]$.

(C) $\forall x \exists! y,[d(x,y) \wedge r(x,y) \rightarrow c(x,y)]$. (D) $\exists x \forall y,[(d(x,y) \wedge r(x,y)) \wedge \neg c(x,y)]$. (E)None of the above.

17. Which of the following statements are true?

(A) Any well-ordered set is also a total-order set. (B) Any well-ordered set is also a partial-order set. (C)Any partial-order set is also a total-order set. (D) Mathematical induction can be applied on any total-order set. (E) A total-order set of predicates is necessary for applying mathematical induction.

18. Which of the following equations about binomial coefficients are always true?($\forall r,s \in R, k,n,m \in N$)

(A) $\binom{r}{k}=\binom{r}{r-k}$. (B) $[\sum_{0\le k\le n}\binom{r}{k}\binom{s}{n-k}]=\binom{r+s}{n}$. (C) $\binom{r}{k}=\binom{r-1}{k}+\binom{r-1}{k-1}$.

(D) $[\sum_{0\le k\le n}\binom{r+k}{k}]=\binom{r+n+1}{n}$. (E) $[\sum_{0\le k\le n}\binom{k}{m}]=\binom{n+1}{m+1}$.

19. An international company use the following procedure to select one employee of the year, where m is input size(the number of employee being considered): (1) if $m<36$, use $(m-1)$ comparison steps to select 1 candidate. Otherwise: (2) use $\lceil \sqrt{m} \rceil$ comparison steps to evenly partition m into 36 subsets(sizes of different subsets differ at most 1); (3)randomly select 6 out of the 36 subsets, and for each subsets follow the same procedure from (1) to select 1 candidate;(4) use 5 comparison steps to select 1 result out of the 6 candidates from each subset. Let $f_{(n)}$ be the number of comparison steps with respect to the input size n, Which of the following are true?

(A) $f(n)=6f(n/36)+\theta(\sqrt{n})$. (B) $f(n)=6f(n/36)+\theta(1)$. (C) $f(n)=\theta(\sqrt{n})$. (D) $f(n)=\theta(n\log n)$.

(E) $f(n)=\theta(n^{1/2}\log n)$.

20. When solving recurrence relation: "$a_n=5a_{n-1}-6a_{n-2}+2, n\geq 2, a_0=3, a_1=7$" using generating function $f(z)$, which of the following are true?

(A) $f(z)=\frac{2z^2}{1-5z+6z^2}$. (B) $f(z)=\frac{2}{1-z}+\frac{1}{1-2z}+\frac{3}{1-3z}$. (C) $f(z)=\frac{1}{1-z}+\frac{2}{1-3z}$. (D) $a_n=2(3^n)+1$.

(E) $a_n=3^{n+1}+2^n+2$.