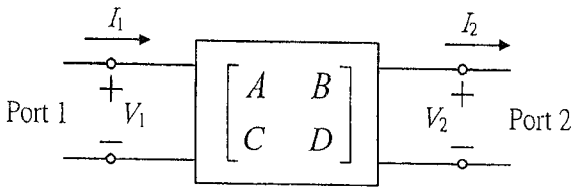
國立中央大學98學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 電波組(一般生)　科目：工程數學　共 2 頁 第 1 頁

＊請在試卷答案卷（卡）內作答

1. (20%) In practical microwave networks, the transmission (or called *ABCD*) matrix is very useful in calculating the overall response of a network with cascaded two-port components. The transmission matrix is defined for a two-port network in terms of the total voltages and currents.



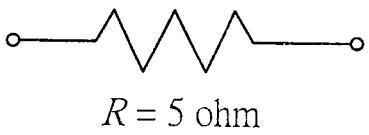
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Therefore, we have that

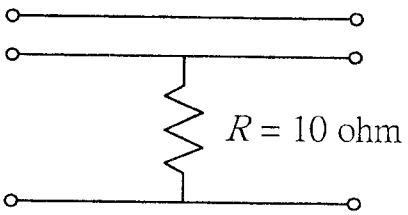
$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$, which indicates that A is found by applying a voltage V_1 at port 1, and measuring the open-circuited voltage V_2 at port 2.

Determine the transmission matrix of the following two-port network,

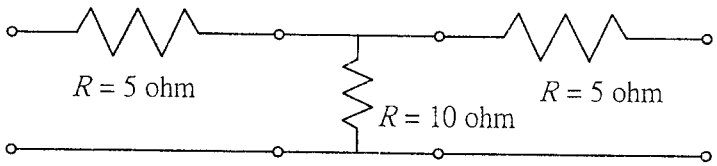
(I)



(II)



(III)



where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{III} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{I} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{II} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{I}$

注意：背面有試題

2. (20%) For a coaxial line with inner conductor radius = a and outer conductor radius = b, a scalar function $V(r,\phi)$ is the solution to Laplace's equation. Solve the boundary value problem

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho\frac{\partial V(r,\phi)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V(r,\phi)}{\partial \phi^2}=0$$
$$V(a,\phi)=V_0, V(b,\phi)=0$$

3. (20%) Determine $u(10,5)$, where

$$\frac{\partial^2 u(x,y)}{\partial x^2}+\frac{\partial^2 u(x,y)}{\partial y^2}=0$$
$$u(x,0)=u(x,10)=0, \quad 0<x<\infty$$
$$u(0,y)=200, \quad 0<y<10$$
$$u(x,y) \text{ is bounded as } x\to\infty$$

4. (20%) Find the linear fractional transform $w=T(z)$ that maps $z_1=-i$, $z_2=0$, $z_3=i$, onto $w_1=-i$, $w_2=-1$, $w_3=i$, respectively. Plot the images of $x=0$, $x=1$, $y=1$, and $y=-1$ under this mapping.

5. (20%) Evaluate the integral $\oint_C \left(\frac{ze^{\pi z}}{z^4-81}+z^2e^{\pi/z}\right)dz$, where C is the ellipse $16x^2+y^2=16$ and oriented counterclockwise.

參考用

注意：背面有試題