## 國立中央大學九十一學年度碩士班研究生入學試題卷

- 1.  $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$  please compute  $A^3 7A^2 + 11A 4I$  detailedly (10%)
- 2. (a) If  $\lambda$  is an eigenvalue of an invertible matrix  $A \cdot x$  is the eigenvector corresponding to  $\lambda$ , prove that  $\frac{1}{\lambda}$  and x are the eigenvalue of  $A^{-1}$  and its corresponding eigenvector respectively. (5%)

(b) If  $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ , show the result of (a) by using any one eigenvalue of A. (5%)

- 3. (a) Suppose [x, y, z]=xi+yj+zk denotes a vector function, where x, y, z are Cartesian coordinates. If we have a function  $f(x,y,z)=2x^2+3y^2+z^2$ , find its directional derivative at the point P:(2,1,3) in the direction of the vector v=i-2k, and then explain the mathematic meaning of the above result. (10%) (b) Using the gradient of  $f(x,y,z)=2x^2+3y^2+z^2$  to find the divergence of grad f. (5%)
- 4. Solve the differential equation  $4x^2y'' + 4xy' y = \frac{12}{x} \quad (10\%)$
- 5. Find the inverse transform of the function

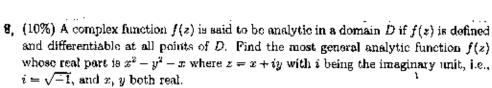
$$\ln(1+\frac{\omega^2}{s^2}) - (7\%)$$

6. Solve the integral equation

$$y(t) = t + \int_0^t \sin(t - \tau) y(\tau) d\tau$$
 (8%)

7. Find the cosine half-range expansion of the function f(x).

$$f(x) = \begin{cases} \frac{2k}{L}x, & \text{if } (0 < x < \frac{L}{2}) \\ \frac{2k}{L}(L - x), & \text{if } (\frac{L}{2} < x < L) \end{cases}$$
(10%)



- **9.** (10%) Let  $f(z) = (z z_0)^m$  be a complex function where m is an integer and  $z_0$  a complex constant. Integrate counterclockwise around the circle C of radius  $\rho$  with the center at  $z_0$ , where  $\rho > 0$ .
- 10. (10%) Derive the following real integral:

$$\int_0^\infty \frac{1}{1+x^4} dx.$$

(Hint: You may use the residue integration method.)

