國立中央大學八十七學年度碩士班研究生入學試題卷

所別：電機工程研究所 甲乙丙丁組 科目：工程數學 共 1 頁 第 1 頁

1. (15 %) Find the engenvalues and eigenvectors of

(a)

$$A=\begin{bmatrix}9 & 1 & 1\\ 1 & 9 & 1\\ 1 & 1 & 9\end{bmatrix},$$

(b)

$$A=\begin{bmatrix}-1 & 0.5\\ -2 & -3\end{bmatrix}.$$

2. (15 %) Find a general solution of

$$y' = Ay + g = \begin{bmatrix}-3 & 1\\ 1 & -3\end{bmatrix} y + \begin{bmatrix}-6\\ 2\end{bmatrix} e^{-2t}$$

3. (10 %) Prove the following formula: A second order differential equation

$$y'' + p(x)y' + q(x)y = r(x) \qquad (1)$$

with arbitrary variable functions $p(x)$, $q(x)$, and $r(x)$ that are continuous on some interval I. A particular solution $y_p(x)$ of (1) on I can be gotten in the form

$$y_p(x) = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx,$$

where W is the Wronskian of y_1, y_2, and y_1, y_2 form a basis of the solutions of the homogenous equation $y'' + p(x)y' + q(x)y = 0$.

4. (10 %) Please find the Laplace transform of the following function

$$g(t) = \begin{cases} 0, & \text{if } t < 3 \\ t^2, & \text{if } t \geq 3 \end{cases}$$

5. (15 %) The dynamic differential equation of a damped forced oscillation spring is $x'' + 2x' + 2x = 20\cos 2t$, with initial condition $x(0) = x'(0) = 0$. Please find its transient solution and steady periodic solution.

(6) (12%, 3% each) Integrate

$$f(z) = \frac{z^2 + 4z + 3}{z^2 - 1}$$

in the clockwise sense around a circle of radius 1 with center at the point

(a) $z = 1$ (b) $z = \frac{1}{2}$ (c) $z = -1 + \frac{1}{2}i$ (d) $z = i$

with i being the imaginary unit, i.e., $i = \sqrt{-1}$.

(7) (11%) Let $P(\mathcal{A})$ denote the probability of event $\mathcal{A}$. For events $\mathcal{A}, \mathcal{B}, \mathcal{C}$, solve the following questions:

(a) (3%) If $P(\mathcal{A}) = 0.2$, $P(\mathcal{A} \cup \mathcal{B}) = 0.5$, and $P(\mathcal{A}|\mathcal{B}) = 0.5$, find $P(\mathcal{B})$.

(b) (4%) If $\mathcal{A}$ and $\mathcal{B}$ are mutually exclusive and independent *and* if $\mathcal{B} \subset \mathcal{A}$, find $P(\mathcal{B})$.

(c) (4%) Show that $2P(\mathcal{A} \cap \mathcal{B}) \leq P(\mathcal{A}) + P(\mathcal{B})$.

(8) (12%, 6% each) Derive the following real integrals:

(a) $\int_0^{2\pi} \frac{1}{\sqrt{2} - \cos\theta} d\theta$ (b) $\int_{-\infty}^{\infty} \frac{\cos(sx)}{k^2 + x^2} dx \quad (s > 0, k > 0)$.

(Hint: You may use the residue integration method)