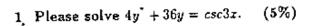
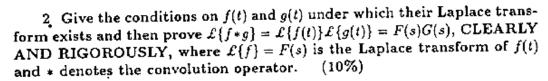
國立中央大學八十四學年度碩士班研究生入學試題卷

所別: 電機工程研究所 組 科目: 工程數學

共2頁第1頁

クランド





3. Suppose that a hawk P at the point (a,0) spots a pigeon Q at the origin flying along the positive y-axis at speed v. The hawk immediately flies toward the pigeon at a speed w.

(a) What will be the flight path of the hawk if w > v? (10%)

(b) If w > v, can the hawk catch the pigeon? where can it do that? (5%)

(c) If $w = v_i$, what will be the flight path of the hawk? (5%)

(Hint: $\int \frac{du}{\sqrt{u^2+v^2}} = \ln(u+\sqrt{u^2+v^2}) + c$).

4. (a) Find two unit vectors orthogonal to u = 3i + 4j - 2k and v = -3i + 4j + k. (5%)
(b) If the following matrix is invertible, find the inverse. (5%)

$$\begin{pmatrix}
1 & -3 & 0 & -2 \\
3 & -12 & -2 & -6 \\
-2 & 10 & 2 & 5 \\
-1 & 6 & 1 & 3
\end{pmatrix}$$

5. (a) Find the rank, the nullity and the kernel of the matrix

$$\begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{pmatrix}. \tag{5\%}$$

(b) Describe geometrically the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T \mathbf{x} = A\mathbf{x}$.

$$A = \begin{pmatrix} \cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{pmatrix}. \quad (5\%)$$

6. Diagonalize
$$\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$
 (10%)

國立中央大學八十四學年度碩士班研究生入學試題卷

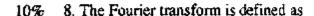
所別:電機工程研究所 組 科目:工程數學 共2頁第2頁

15% 7.

5% a/ Show that $\cosh z = \cosh x \cos y + i \sinh x \sin y$, where z=x+iy.

5% b/ Show that $\sin^{-1} z = -i \ln \{iz \pm (1-z^2)^{1/2}\}.$

5% c/ Find the principle value of (1+i)1-i.



$$F(w) = \int_{-\infty}^{\infty} f(t)e^{-iwt} dt, \quad -\infty < w < \infty$$

Find the Fourier transform of the following function:

$$f(t) = \frac{i}{4+t^2}$$

10% 9. The Laplace transform is defined as

$$F(s) = \int_0^\infty f(t)e^{-st} dt$$

Use residue theorem to find the inverse Laplace transform f(t) of the following:

$$F(s) = \frac{s}{s^2 + 1}$$

