國立中央大學 107 學年度碩士班考試入學試題

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科目：　工程數學(不含複變)

本科考試禁用計算器

＊請在答案卷(卡)內作答

1. **(20%)** Find a matrix A such that $A^2 = \begin{bmatrix} 9 & -5 & 3 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

2. **(20%)** Let $E = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = [(1, 1, 1)^T, (2, 3, 2)^T, (1, 5, 4)^T]$
   $F = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3] = [(1, 1, 0)^T, (1, 2, 0)^T, (1, 2, 1)^T]$.
   If $\mathbf{x} = 8\mathbf{u}_1 - 5\mathbf{u}_2 + 3\mathbf{u}_3$ and $\mathbf{y} = -8\mathbf{u}_1 + 2\mathbf{u}_2 + 3\mathbf{u}_3$,
   find the coordinates of **x** and **y** with respect to the ordered basis E.

3. **(20%)** Find the Inverse **Laplace transform** of the following functions:

   (a) $cot^{-1}(\frac{s}{w})$ **(10%)**

   (b) $\ln\frac{s+a}{s+b}$ **(10%)**

4. **(20%)** Solve the following differential equation:

$$x^2y'' + xy' - y = x^3e^x$$

5. **(10%)** For two continuous-time signals $x(t)$ and $h(t)$, the $x(t)$ is a repetitive impulse sequence with interval T=2, and $h(t) = e^{-2t}u(t)$ is a decaying exponential signal with finite length from $t = 0$ to $t = 1$ (see Fig. 1). Please find the Fourier representation of their convolution output $y(t) = x(t) * h(t)$, where the $*$ is the continuous-time convolution operator.

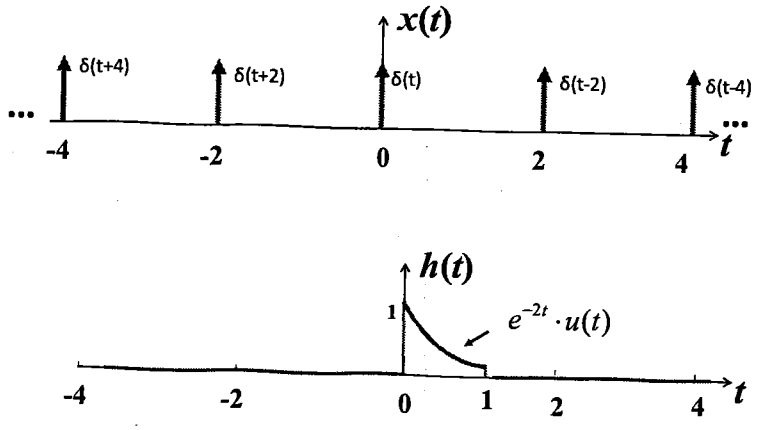

Figure 1.

注意:背面有試題

(a) **(2%)** Is $y(t)$ a periodic signal or non-periodic signal?

(b) **(8%)** Please determine the coefficients of Fourier series for $y(t)$, if $y(t)$ is a periodic signal. Otherwise, please determine the complex Fourier transform for $y(t)$, if $y(t)$ is a non-periodic signal.

6. **(10%)** For two continuous-time non-periodic signals, $x(t)$ and $y(t)$, please prove the following Fourier properties:

(a) **(5%)** $F\{x(t) * y(t)\} = X(j\omega) \cdot Y(j\omega)$.

(b) **(5%)** $F\{-jt \cdot x(t)\} = \dfrac{d}{d\omega} X(j\omega)$

Remark: $F\{\ \}$ is continuous-time Fourier transform operator, and $X(j\omega)$ and $Y(j\omega)$ are Fourier transform of $x(t)$ and $y(t)$, respectively.