國立中央大學103學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 系統與生醫組(一般生)　科目：信號與系統　共 2 頁 第 1 頁

本科考試禁用計算器　＊請在試卷答案卷（卡）內作答

一、**(20%) Figure 1** is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a low-pass differentiator. For each of the input signals $x(t)$ below, determine the filtered output signal $y(t)$.

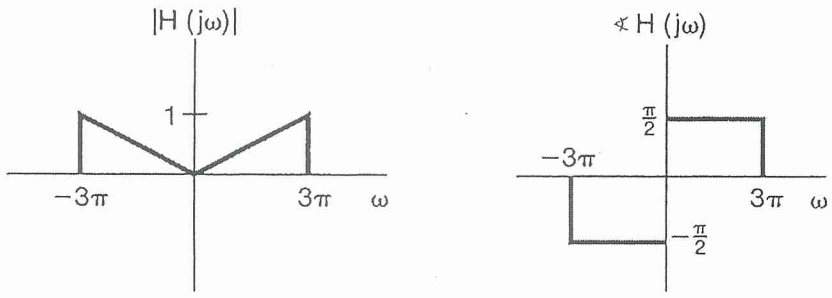

Figure 1.

(一) (10%) $X(j\omega)=\dfrac{1}{2+j\omega}$.

(二) (10%) $x(t)=\cos(4\pi t+\theta)$.

二、**(10%)** Consider the signal

$$x(t)=\left(\frac{\sin 50\pi t}{\pi t}\right)^2,$$

which we wish to sample with a sampling frequency of $\omega_s=150\pi$ to obtain a signal $g(t)$ with Fourier transform $G(j\omega)$. Determine the maximum value of ω_0 for which it is guaranteed that

$$G(j\omega)=75X(j\omega)\text{for}|\omega|\le\omega_0,$$

where $X(j\omega)$ is the Fourier transform $x(t)$.

三、**(10%)** How many signals have a Laplace transform that may be expressed as

$$\frac{(s-1)}{(s+2)(s+3)(s^2+s+1)}$$

in its region of convergence?

四、**(10 %)** The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt}+10y(t)=\int_{-\infty}^{+\infty}x(\tau)z(t-\tau)d\tau-x(t),$$

where $z(t)=e^{-t}u(t)+3\delta(t)$.

(一) (5%) Find the frequency response $H(j\omega)=Y(j\omega)/X(j\omega)$ of this system.

(二) (5%) Determine the impulse response of the system.

注意：背面有試題

五、**(15%)** For a causal linear time-invariant (LTI) system, it has impulse response $h[n]=n(0.5)^n u[n]$. Please use Z-transform to find its output $y[n]=x[n]*h[n]$ when the input signal $x[n]$ is $x[n]=(0.4)^n u[n]$, where * is the time-domain convolution operator and $u[n]$ is the unit step function.

六、**(15 %)** Please prove the following discrete-time Fourier transform (DTFT) properties:

(一) (5%) $x[n]\cdot y[n] \xrightarrow{DTFT} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\Omega-\theta)})d\theta$.

(二) (5%) $x[-n] \xrightarrow{DTFT} X(e^{-j\Omega})$.

(三) (5%) $e^{j\Omega_0 n}x[n] \xrightarrow{DTFT} X(e^{j(\Omega-\Omega_0)})$.

※Remark: n is the discrete-time index, Ω is the radial frequency, * is discrete-time convolution operator, and $X(e^{j\Omega})$ and $Y(e^{j\Omega})$ are the discrete-time Fourier transforms of discrete-time sequences $x[n]$ and $y[n]$, respectively.

七、**(20%)** Consider a signal flow graph shown in **Figure 2**.

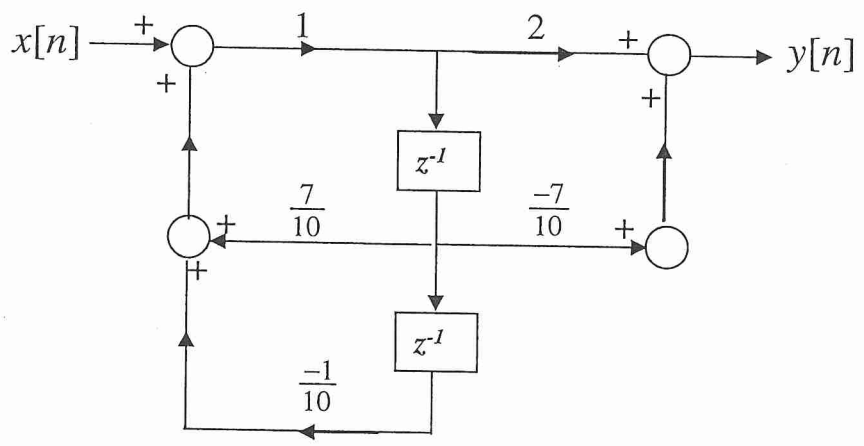

Figure 2.

(一) (7%) Please find the transfer function $H(z)$.

(二) (7%) If the system is causal, please find the impulse response $h[n]$ of the system.

(三) (6%) Please derive the Fourier transform $H(e^{j\Omega})$ of $h[n]$ from $H(z)$.