科目：工程數學A(5003)

校系所組：中央大學電機工程學系(電子組、系統與生醫組)
交通大學電子研究所(甲組、乙組)
清華大學電機工程學系(甲組)
清華大學光電工程研究所
清華大學電子工程研究所
清華大學工程與系統科學系（丁組）
陽明大學醫學工程研究所（醫學電子組）

1. (13%) For a 3x3 matrix A, where $A=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & -2\\ 2 & -1 & -1\end{bmatrix}$

(a) Please find its eigenvalues and corresponding eigenvectors. (6 %)
(b) Assume the 3 eigenvalues are in the order of $\lambda_1 \le \lambda_2 \le \lambda_3$. Starting from the eigenvector corresponding to λ_1, please find the corresponding orthonormal basis. (7 %)

2. (12%) Define the space P_n as the set of all polynomials of degree less than n. Let L be the operator on P_3 and

$$L(p(x)) = \alpha + x\frac{d}{dx}p(x) + \frac{d^2}{dx^2}p(x)$$

(a) (3%) Find the matrix **A** representing L with respect to $[1, x, x^2]$.
(b) (3%) Find the matrix **B** representing L with respect to $[1, x, 1+x^2]$.
(c) (3%) Find the condition of α such that A and B are similar matrices.
(d) (3%) If $p(x) = a_0 + a_1x + a_2x^2$, calculate $L^n(p(x))$ given the condition of α in (c).

3. (7%) Develop $f(z)=\frac{1}{1-z^4}$ in a Maclaurin series and find the radius of convergence.

4. (8%) Evaluate $\int_C \bar{z}dz$, where C is from 0 along the parabola $y = x^2$ to $3 + 9i$.

5. (10 %) Calculate $\int_{-\infty}^{\infty}\frac{e^{ax}}{1+x}dx$, where $0 < a < 1$.

6. (2%) (a) Solve $y' = y^2, y(0) = 2$. Call the solution y_c.
(2%) (b) Solve $y' = y^2 - 1, y(0) = 1$. Call the solution y_p.
(1%) (c) Does y_c+y_p solve $y' = y^2 - 1, y(0) = 3$? Explain.

7. (8%) One solution of the equation $y'' + p(t)y' + q(t)y = 0$ is $(1+t)^2$, and the Wronskian of any two solutions is constant. Find the general solution of $y'' + p(t)y' + q(t)y = 1 + t$.

8. (5%) Three solutions of a 2nd-order linear equation $L(y) = g(t)$ are

$$\psi_1 = 2e^{t^2} + e^t, \psi_2 = te^{t^2} + e^t \text{ and } \psi_3 = (1+t)e^{t^2} + e^t.$$

Find the solution of the initial problem $L(y) = g(t); y(0) = 3, y'(0) = 0$

注意:背面有試題

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9. (8%) Let y be a real function of x. Find two linearly independent Frobenius solutions of the following differential equation at $x = 0$:

$$2x^2y''+x(x-3)y'+3y=0$$

10. (8%) Let x_1 and x_2 be two real functions of t. Solve x_1 and x_2 for the following system of differential equations

$$\begin{cases} x_1' = 4x_1 - x_2 \\ x_2' = x_1 + 2x_2 \end{cases}, x_1(1)=5, x_2(1)=3$$

11. (7%) Given the initial value problem, $x''+4x'+13x=f(t);\ x(0)=x'(0)=0$
(a) (3%) Express $x(t)$ in terms of $f(t)$ and convolution.
(b) (4%) Solve $x(t)$ for $f(t)=u(t)-u(t-1)$, where $u(t)$ denotes the unit step (or Heaviside Step) function.

12. (9%) $f(t)=\begin{cases}1, & 0<t<5\\ 0, & 5<t<10\end{cases}$ with $f(t+10)=f(t)$ is a piecewise continuous and periodic function that satisfies $f(t)=\dfrac{[f(t^+)+f(t^-)]}{2}$, where $f(t^+)$ and $f(t^-)$ are the right-hand and left-hand limits of $f(t)$ at each discontinuity.
(c) (3%) Find the Fourier series of $f(t)$.
(d) (3%) Let $f(t)$ be defined for $t \geq 0$; find its Laplace transform $F(s)$ for $s>0$.
(e) (3%) Find a particular solution for $x''+16x=f(t)$.