國立中央大學102學年度碩士班考試入學試題卷

所別：數學系碩士班 乙組(一般生)　科目：數值分析　共 2 頁 第 1 頁
本科考試禁用計算器

*請在試卷答案卷（卡）內作答

參考用

Instructions: Do all 4 problems. Show your work.

1. **(Computer arithmetics)**
   Use the following example
   $$f(x) = \sqrt{x^2+1} - 1,$$
   whose value needs to be evaluated for x near zero, to explain what the **loss of significance** means and propose a way to avoid it in substraction. (10 pts)

2. **(Numerical linear algebra)**
   Consider the matrix A given by
   $$\begin{pmatrix} 0 & 2 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{pmatrix}$$
   (a) Find the LU decomposition of PA, where P is a permutation matrix. (10 pts)
   (b) Use Gershgorin's Theorem to locate the eigenvalues of A. (5 pts)
   (c) Give a bound for the spectral radius, $\rho(A)$. (5 pts)
   (d) Design an algorithm based on the Power method for finding the second largest eigenvalue in magnitude. (10 pts)

3. **(Interpolation)**
   Consider the table

x	1	4	2	3
y	-1	-3	-2	1

   (a) Find the Lagrange form of the interpolation polynomial of degree 3 passing the points given in the above table. (10 pts)
   (b) Redo part (a) by using the Newton form of the interpolation polynomial. (10 pts)

注意：背面有試題

4. (Numerical Differential Equations)

(a) Use Taylor's theorem to derive the first-order backward difference approximation for the first derivative of $u(x)$ at $x = t$ and the second-order central difference approximation for the second derivative of $u(x)$ at $x = t$, including the error term, for $h > 0$:

$$u'(t) = \frac{u(t) - u(t-h)}{h} + \frac{h}{2}u''(\xi), \tag{1}$$

where $\xi \in (t-h, t)$. and

$$u''(t) = \frac{u(t+h) - 2u(t) + u(t-h)}{h^2} - \frac{h^2}{12}u^{(4)}(\xi), \tag{2}$$

where $\xi \in (t-h, t+h)$. Be sure to write any assumption you made for the function $f(x)$. (10 pts)

(b) Discretize the following two-point boundary value problem:

$$\begin{cases} -u'' + \sinh(u) = 0 \text{ in } (0,1) \\ u(0) = 1 \qquad u'(1) = 0 \end{cases}$$

using Formulas (1) and (2) with the grid size $h = 1/4$ to obtain the nonlinear system of equations in the form of

$$\begin{cases} f_1(u_1, u_2, u_3) = 0 \\ f_2(u_1, u_2, u_3) = 0 \\ f_3(u_1, u_2, u_3) = 0 \end{cases}$$

excluding two boundary points, x_0 and x_4. Here, u_i are the approximate values of $u(x)$ at interior points x_i for $i = 1, 2, 3$. Note that $\sinh x = \frac{e^x - e^{-x}}{2}$. (10 pts)

(c) Solve the resulting nonlinear system of equations by Newton's method. Perform one Newton iteration starting with $(u_1, u_2, u_3)^T = (0, 0, 0)$. (10 pts) Show that the corresponding Jacobian matrix is always symmetric positive definite. (10 pts)

注意：背面有試題