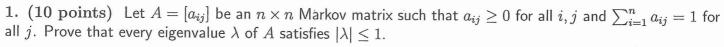
國立中央大學102學年度碩士班考試入學試題卷

所別:<u>數學系碩士班 甲組(一般生)</u> 數學系碩士班 甲組(在職生) 數學系碩士班 乙組(一般生) 科目:線性代數 共___頁 第____頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答



- 2. (10 points) Let A be a non-zero square matrix which satisfies $A^{2013}=0$. Show that A is not diagonalizable.
- 3. (10 points) Let V be a complex inner-product space with inner product $\langle \, , \rangle$. Prove that

$$|\langle v, w \rangle|^2 \le \langle v, v \rangle \langle w, w \rangle$$

for any $v, w \in V$.

4. (10 points) Let A be the matrix

$$\left[\begin{array}{ccccc}
0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 3
\end{array}\right]$$

Find the minimal polynomial of A. Show your work.

- 5. (10 points) Let V be an infinite dimensional vector space. A linear operator $P:V\to V$ is called a projection if $P^2=P$. Prove that if P is a projection, then the eigenvalues of P are either 0 or 1.
- 6. (20 points) Let A, B be square matrices of the same size. Prove that if λ is an eigenvalue of AB, then λ is also an eigenvalue of BA.
- 7. (10 points) Prove or give a counter example: If A and B are 2×2 real matrices, then then minimal polynomial of AB equals the minimal polynomial of BA.
- 8. (20 points) Let A and B be matrices of size 3×2 and 2×3 respectively. Suppose that AB is given by

$$AB = \left[\begin{array}{rrr} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{array} \right].$$

Find the product BA. Show your work.



