所別：數學系碩士班 甲組(一般生)　　科目：線性代數　共 ___1___ 頁 第 ___1___ 頁
　　　　數學系碩士班 甲組(在職生)
　　　　數學系碩士班 乙組(一般生)
本科考試禁用計算器　　　　　　　　　　　　　　　　　　　　　　＊請在試卷答案卷（卡）內作答

1. **(10 points)** Let $A = [a_{ij}]$ be an $n \times n$ Markov matrix such that $a_{ij} \geq 0$ for all i, j and $\sum_{i=1}^{n} a_{ij} = 1$ for all j. Prove that every eigenvalue λ of A satisfies $|\lambda| \leq 1$.

2. **(10 points)** Let A be a non-zero square matrix which satisfies $A^{2013} = 0$. Show that A is not diagonalizable.

3. **(10 points)** Let V be a complex inner-product space with inner product $\langle , \rangle$. Prove that

$$|\langle v, w \rangle|^2 \leq \langle v, v \rangle \langle w, w \rangle$$

for any $v, w \in V$.

4. **(10 points)** Let A be the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

Find the minimal polynomial of A. Show your work.

5. **(10 points)** Let V be an infinite dimensional vector space. A linear operator $P : V \to V$ is called a *projection* if $P^2 = P$. Prove that if P is a projection, then the eigenvalues of P are either 0 or 1.

6. **(20 points)** Let A, B be square matrices of the same size. Prove that if λ is an eigenvalue of AB, then λ is also an eigenvalue of BA.

7. **(10 points)** Prove or give a counter example: If A and B are 2×2 real matrices, then then minimal polynomial of AB equals the minimal polynomial of BA.

8. **(20 points)** Let A and B be matrices of size 3×2 and 2×3 respectively. Suppose that AB is given by

$$AB = \begin{bmatrix} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{bmatrix}.$$

Find the product BA. Show your work.